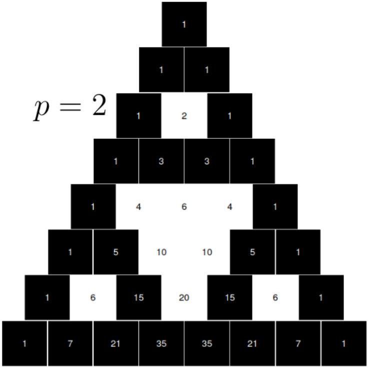
What is...an inverse fractal?

Or: Zooming out

Inverse fractals - discrete self-similarity

No!

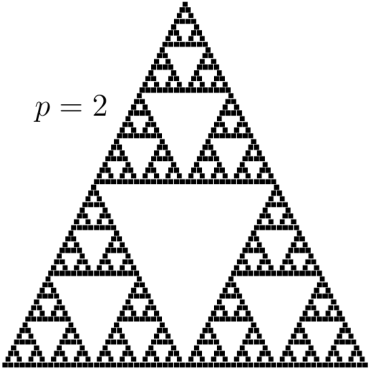
$p = 2$



Yes!

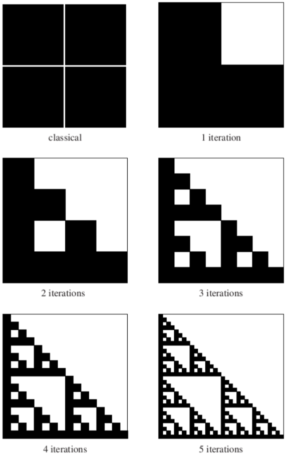
,

$p = 2$



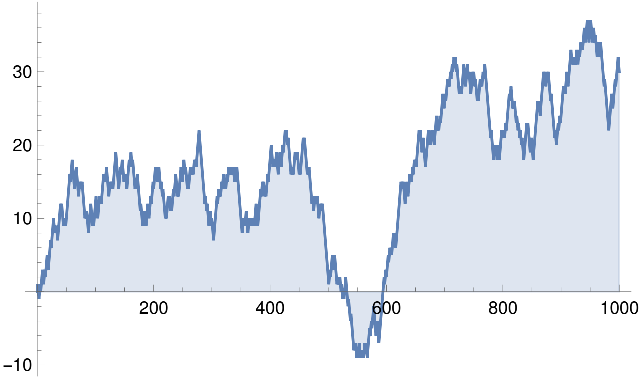
- **Fractal** = something that is self-similar, an object of dynamics
- **Inverse fractal** = the same, but as an object of counting
- **Example** Sierpiński's triangle versus Pascal's triangle mod 2

Divide and conquer



- Let us say we have an algorithm that takes n^2 operations
- Let us say we find a way to safe $1/4$ operations
- Recursion then gives an inverse fractal

Coin toss



- Let us say we toss a coin
- Now write down the graph of heads versus tails
- Almost always this is an inverse fractal

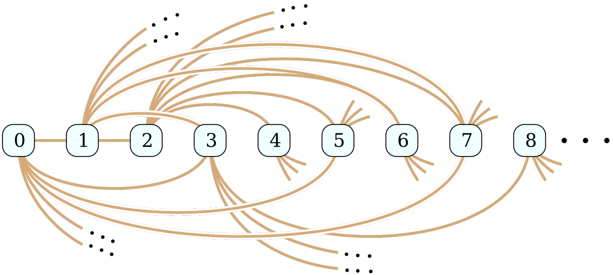
Enter, the theorem

There are three fractal graphs (inverse fractals) on ∞ countably many vertices

(i) The graph without edges (boring!)

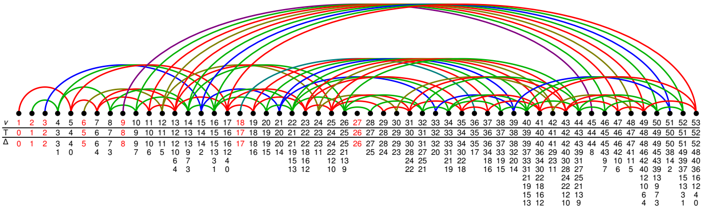
(ii) The complete graph (boring!)

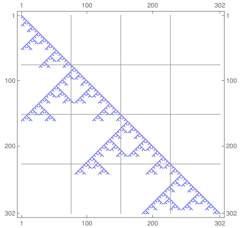
(iii) The Rado graph:



- **Fractal graph** (definition for this video) = every partition of the graph into finitely many parts has some subgraph isomorphic to the original graph
- **Rado graph** = infinite coin flip graph

More inverse fractal in combinatorics





- Zoo of examples Representation theory of $GL_n(\bar{\mathbb{F}}_p)$ and friends
- Zoo of examples Representation theory of symmetric groups over $\bar{\mathbb{F}}_p$
- This is one reason why these two are so difficult

Thank you for your attention!

I hope that was of some help.