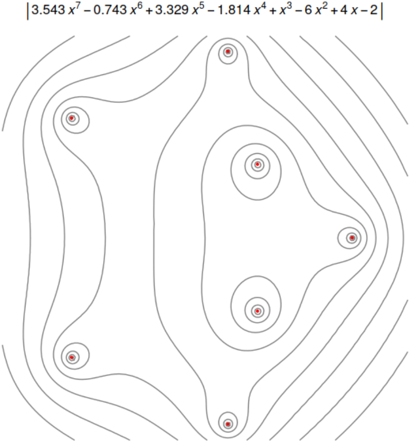
What is...the art of locating roots?

Or: Finding roots without finding them

The fundamental theorem of algebra

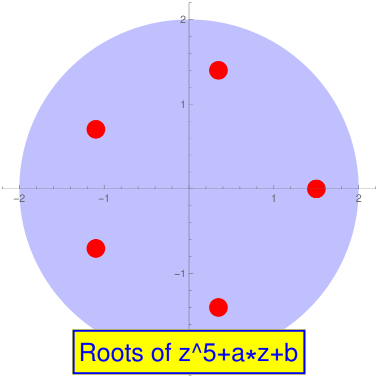


Every polynomial has complex roots, but calculating roots is hard

Question. How can one avoid calculations?

Finding roots à la Rouché

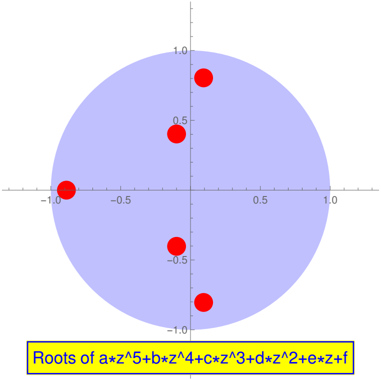
If $|(f(z)+g(z))-f(z)|<|f(z)|$ for all $z \in \delta C$, then f and $f+g$ have the same number of zeros in C f dominates



- $f = z^5$, $g = 4z + 2$
- $|g(z)| < |f(z)|$ for all $|z| < 2$ $f(2) = 32$
- $f + g = z^5 - 4z + 2$ has all of its zeros in the disc of radius 2 Located!

Finding roots à la Eneström–Kakeya

$f = a_n \cdot z^n + \ldots + a_0$ with real $a_n \geq \ldots \geq a_0 \geq 0$ Non-negative with order



- $f = 10 \cdot z^5 + 9 \cdot z^4 + 8 \cdot z^3 + 8 \cdot z^2 + 2 \cdot z + 1$
- $10 \geq 9 \geq 8 \geq 8 \geq 2 \geq 1$ Order!
- f has all of its zeros in the disc of radius 1 Located!

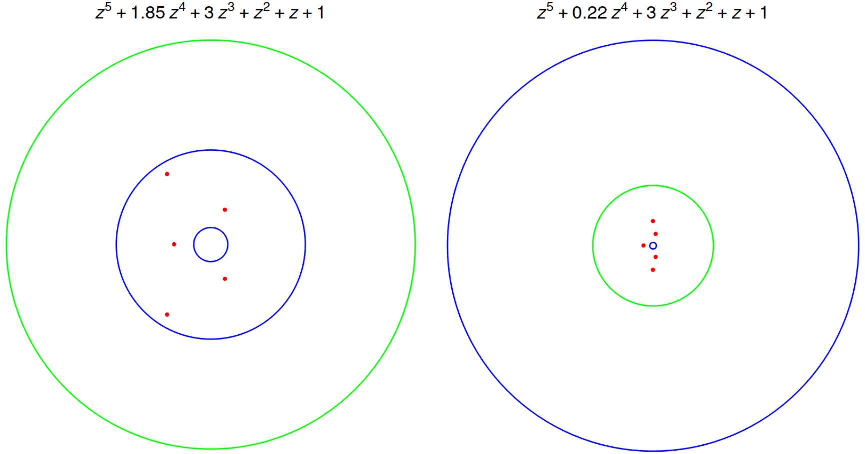
Enter, the theorems/philosophy!

Do not calculate roots – locate them!

Some examples

- Rouché With dominating f
- Original Eneström–Kakeya Non-negative with order
- Strengthened Eneström–Kakeya: f with coefficients $a_j \in \mathbb{R}_{\geq 0}$ has roots in an annulus $\min a_j/a_{j+1} \leq |z| \leq \max a_j/a_{j+1}$ Non-negative real
- Reversed Eneström–Kakeya: f with real $a_0 \geq \ldots \geq a_n \geq 0$ has roots in $1 \leq |z|$ Non-negative with order
- Joyal–Labelle–Rahman: f with real $a_0 \geq \ldots \geq a_n$ has roots in $|z| \leq (a_n - a_0 + |a_0|)/|a_n|$ With order
- Many more formulas can be found in the link in the description

Rouché (green, $|z| \leq \max a_i + 1$) vs. strengthened Eneström–Kakeya (blue)



The efficiency of location depends on the polynomial

Thank you for your attention!

I hope that was of some help.