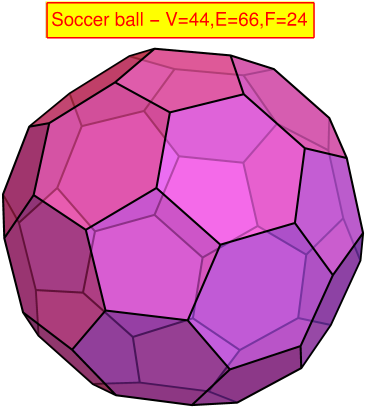
What is...Euler's polyhedron formula?

Or: 2000 years of not doing the count.

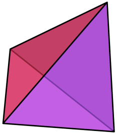
The soccer ball



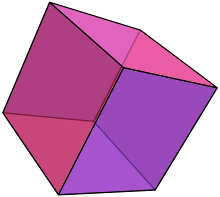
Random(?) fact. We have $V - E + F = 2$ “vertices - edges + faces = 2”

The platonic solids – and Euler counted

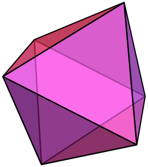
Tetrahedron – V=4,E=6,F=4



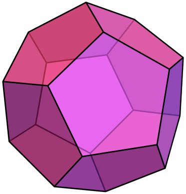
Cube – V=8,E=12,F=6



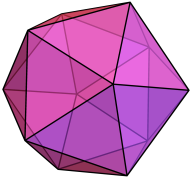
Octahedron – V=6,E=12,F=8



Dodecahedron – V=20,E=30,F=12



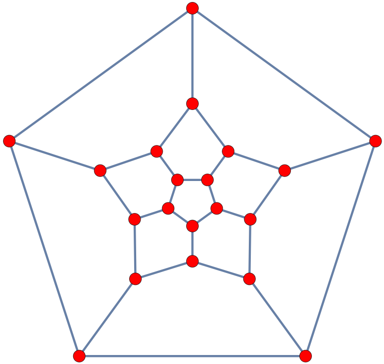
Icosahedron – V=12,E=30,F=20



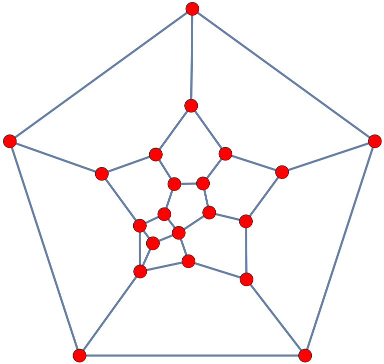
Euler's observation. We still have $V - E + F = 2$

For connected plane graphs we have $V - E + F = 1$:

Plane graph 1 – V=20,E=30,F=11



Plane graph 2 – V=20+1,E=30+3,F=11+2



Proof? Induction (as illustrated)

Why $= 1$ and not $= 2$? Well, there is an outside face which is not counted

Enter, the theorem!

For any spherical polyhedron we have $V - E + F = 2$

There are dozens of known proofs – *e.g.* use plane graphs via stereographic projection

Matt Parker's (youtube: standupmaths) petition

Soccer signs in the UK - only hexagons:

This is impossible:

$$V - E + F = 2F + 3F - F = 0 \stackrel{!}{=} 2$$

Thank you for your attention!

I hope that was of some help.