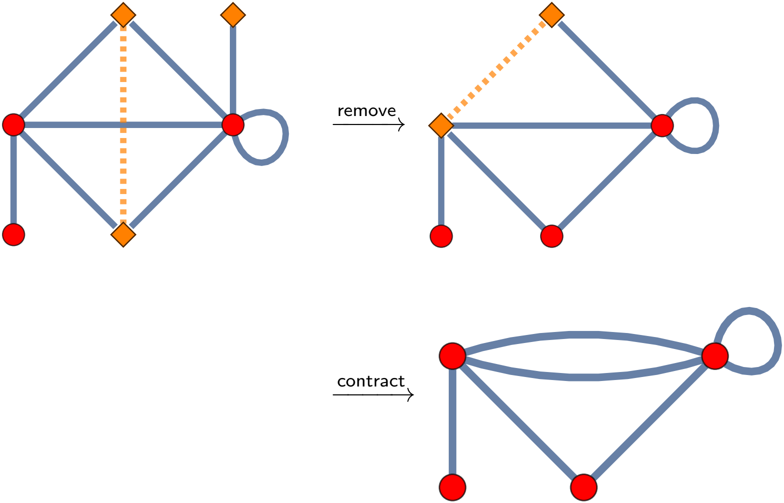
What is...the Robertson–Seymour theorem?

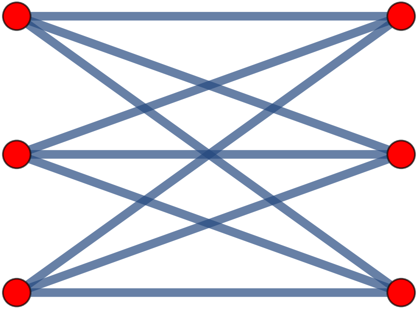
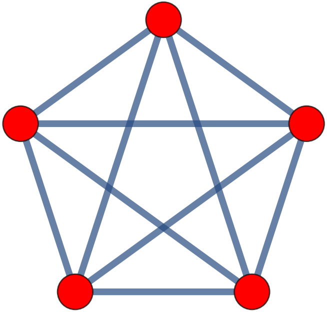
Or: Minors are majors

Generalized substructures



- G has H as a minor, if H is obtained from G via remove & contract
- Minors are like subgraphs, but more general

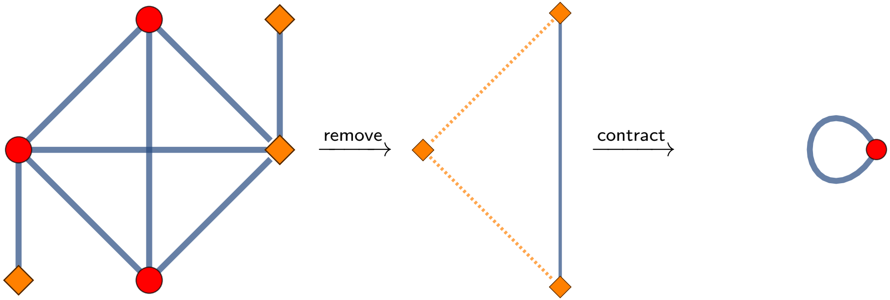
Kuratowski–Wagner's theorem

$K_{3,3} =$  , $K_5 =$ 

- Planar (=we can draw them in the plane) graphs are minor closed
- A graph is planar $\Leftrightarrow$ it does not contain $K_{3,3}$ and K_5 as minors
- There is a finite list of forbidden graphs, namely $K_{3,3}$ and K_5

Trees/forests and minors

loop = 



- Forest (=no circles) graphs are minor closed
- A graph is a forest ⇔ it does not contain a loop as a minor
- There is a finite list of forbidden graphs, namely a loop

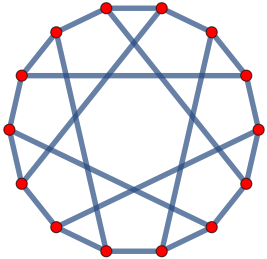
Enter, the theorem

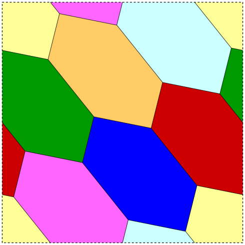
For every minor-closed family of graphs, the set of forbidden minors is finite

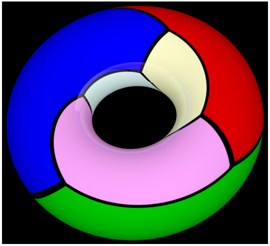
- A class M of graphs is minor-closed if for every $G \in M$, all minors of G are also in M
- Every minor-closed class M can be described by specifying the set of all minor-minimal graphs that are not in M
- These minor-minimal graphs are called forbidden minors Obstructions
- This theorem was proved it in a series of twenty papers spanning over 500 pages from 1983 to 2004

Equivalent is the powerful theorem:
For every minor-closed family M of graphs there exists a cubic time algorithm $\mathcal{O}(n^3)$ for testing membership in M: one simply checks if a given graph contains some forbidden minor for M

Embedding into tori? Well...







- Being toroidal is minor closed, so there is a finite list of obstructions
- There are at least 17535 obstructions, the precise number is unknown in 2021

Thank you for your attention!

I hope that was of some help.