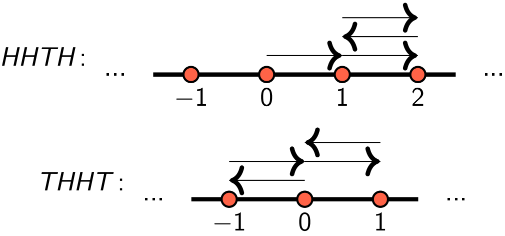
What is...Polya's theorem?

Or: Birds get lost

The coin flip experiment on a line

- Fix $\mathbb{Z}$ as our underlying world
- Flip a coin and move along $\mathbb{Z}$ by $+1$ for heads and -1 for tails Random



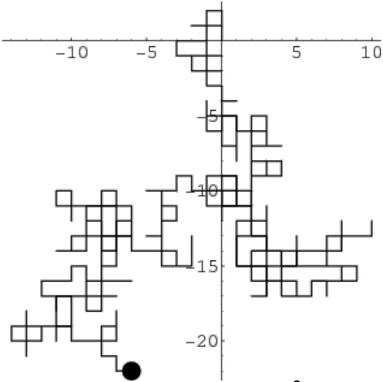
Question Can we (in some sense) predict what happens?

You always come home

(From 01.Jan.99 to 05.Dec.14 $\rightarrow$ 4006 observations)

- Expected distance from origin $\rightarrow \sqrt{n}$ Arbitrary far away from home
- A random walk will cross the origin eventually A 1d walker will return home

You always come home – even in $\mathbb{Z}^2$



Random walk on $\mathbb{Z}^2$

- Expected distance from origin $\to \sqrt{n}$ Arbitrary far away from home
- A random walk will cross the origin eventually A 2d walker will return home

Enter, the theorem

For random walks on $\mathbb{Z}^d$ we have:

- The expected average distance from the origin is

$$\sim \sqrt{n} \cdot c(d) \text{ where } c(d) = \text{constant depending on } d$$

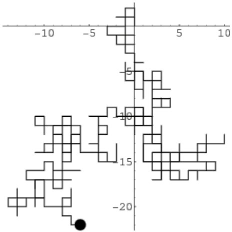
Arbitrary far away from home

- A random walk will cross the origin eventually with probability

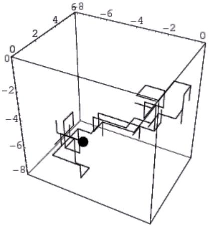
d	1	2	3	4	5	6	7	8
%	1	1	≈0.34	≈0.19	≈0.14	≈0.10	≈0.09	≈0.07

A 3d walker will not necessarily return home

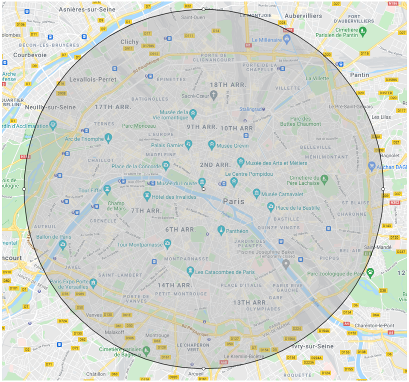
drunk human will return home:



drunk bird might get lost:



Will I find Paris' center?



- Say Paris is 6000m in radius
- Start at Paris' center, get drunk and random walk with step 1m
- You will revisit Paris' center with about 85% chance before you leave Paris

% that a random walk on $\mathbb{Z}^2$ gets more than distance n away from the origin without revisiting it is approximately $\approx \left(1.0293737 + \frac{2}{\pi}\ln(n)\right)^{-1}$

Thank you for your attention!

I hope that was of some help.