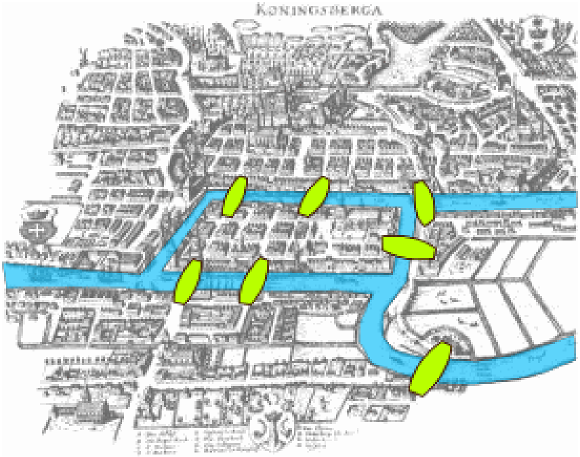
What is...the difference between Eulerian and Hamiltonian graphs?

Or: They are not dual!?

Euler



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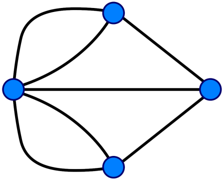
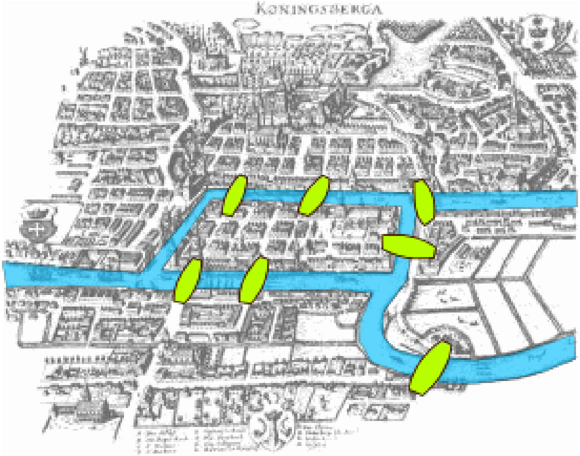


There is no such cycle

- An Eulerian circle in a graph visits every edge exactly once
- There is an easy criterion to decide whether a graph is Eulerian

Hamilton



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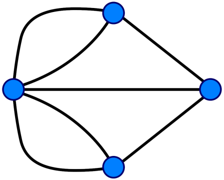
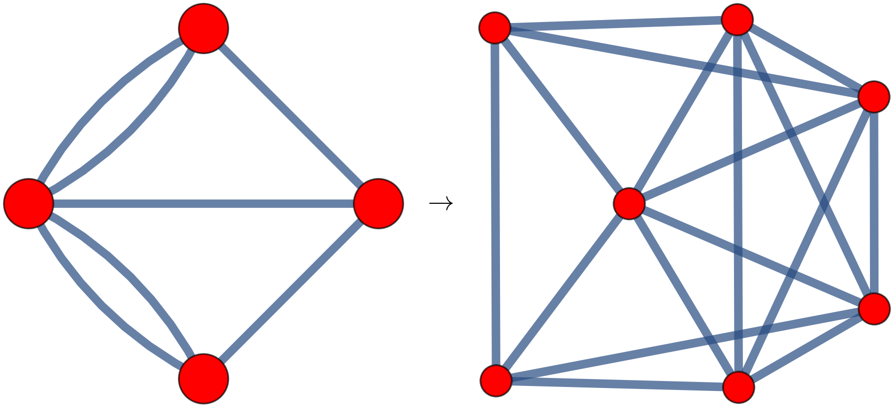


There is such a cycle

- A Hamiltonian circle in a graph visits every vertex exactly once
- There is no known easy criterion to decide whether a graph is Hamiltonian

Wait, aren't these dual problems?



$G \to L(G)$

- These problems are not dual in any known way
- G Eulerian $\Rightarrow$ its line graph $L(G)$ is Hamiltonian
- $L(G)$ Hamiltonian $\not\Rightarrow$ G is Eulerian

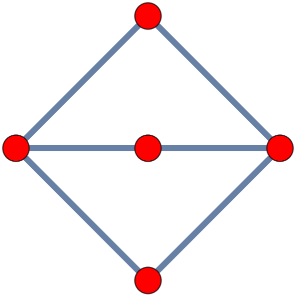
Enter, the theorems

- A graph is Eulerian $\Rightarrow$ Every vertex has even degree **Easy to check**
- There are very effective algorithms to construct Eulerian cycles $< \mathcal{O}(|E|^2)$

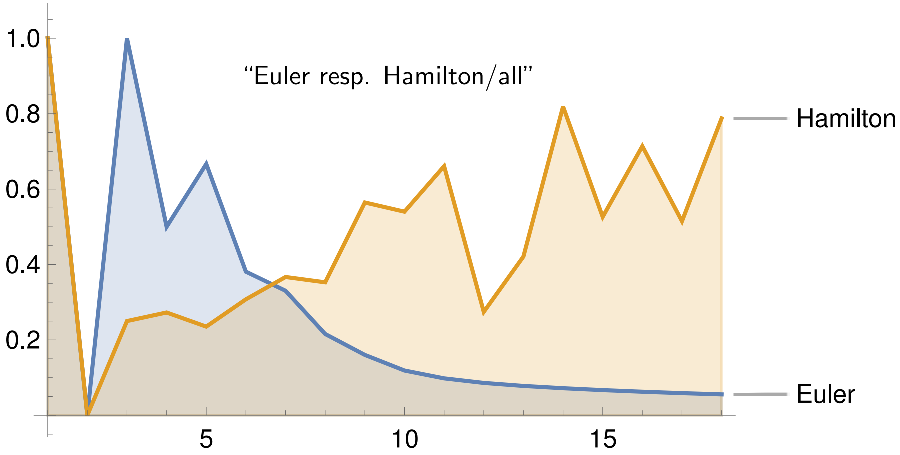
- To determine whether a graph is Hamiltonian is NP-complete **Very hard**
- The algorithms to construct Hamiltonian cycles are “brute force” **Very slow**

Some progress for checking Hamiltonian has been made (Dirac, Ore, Bondy–Chvátal...): usually graphs with few edges tend to be non-Hamiltonian

Non-Hamiltonian:



Everything is Hamiltonian, and everything is complicated



- $G_{n,p}$ – random graph on n vertices with edges put in with probability $p = 1/2$
- Probability of $G_{n,p}$ to be Eulerian $\xrightarrow{n\to\infty} 0$ Nothing
- Probability of $G_{n,p}$ to be Hamiltonian $\xrightarrow{n\to\infty} 1$ Everything

Thank you for your attention!

I hope that was of some help.