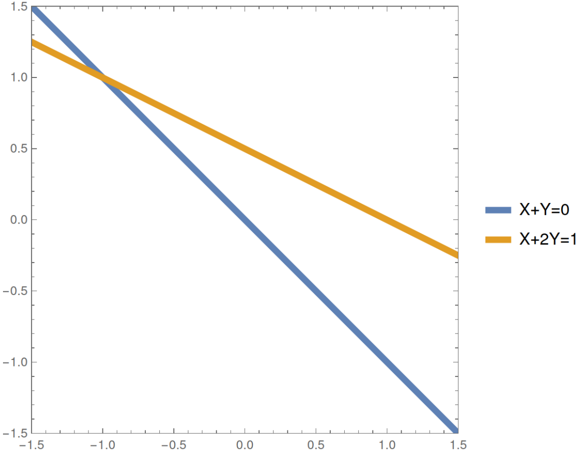
What is...Bézout's theorem?

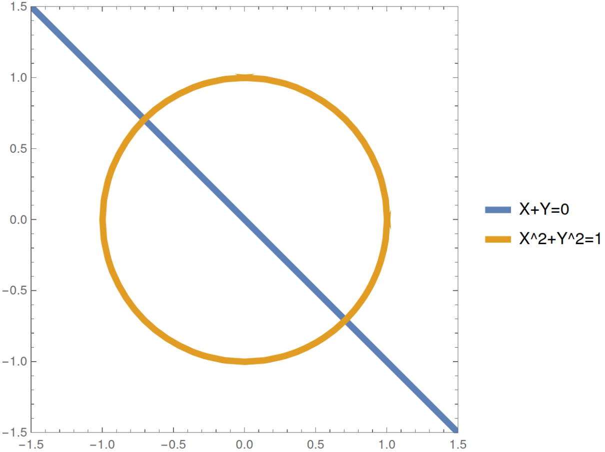
Or: Counting intersections

Degrees 1 and 1



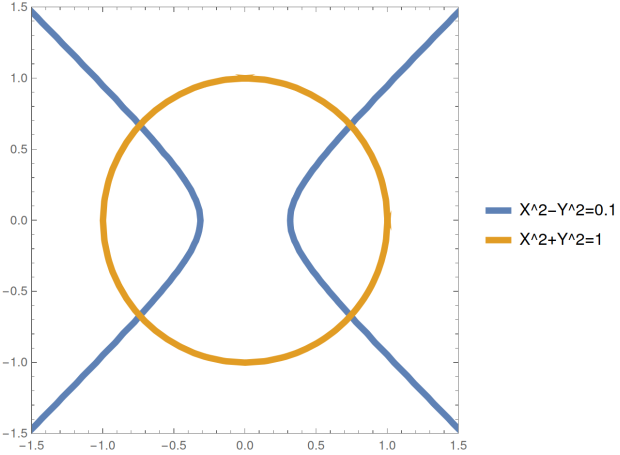
- Line $aX + bY = c$ Line $a'X + b'Y = c'$
- Generically they intersect in one point
- Projectively there are no non-intersections

Degrees 1 and 2



- Line $aX + bY = c$ Circle $aX^2 + bY^2 = c$
- Generically they intersect in two points
- Over $\mathbb{C}$ there are no non-intersections

Degrees 2 and 2



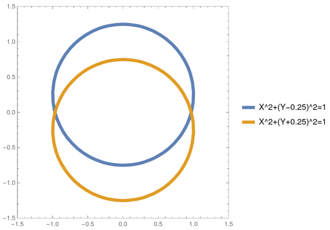
- Degree 2 curve $aX^2+bXY+cY^2=d$ Degree 2 curve $a'X^2+b'XY+c'Y^2=d'$
- Generically they intersect in 4 points
- "Special" intersections are double intersections

Enter, the theorem

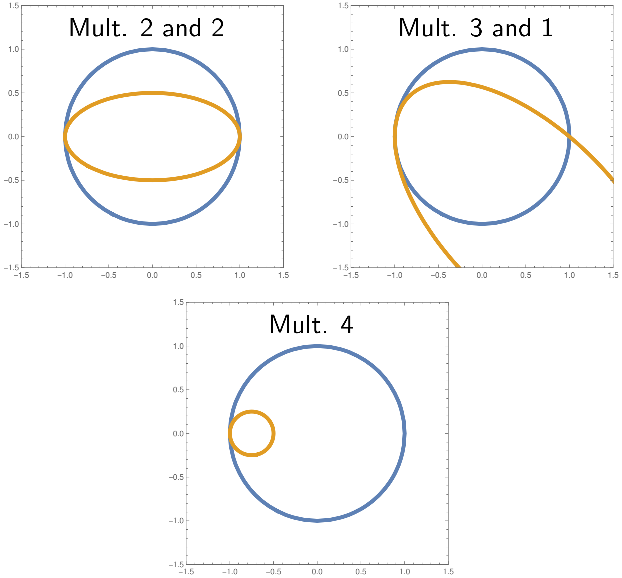
X, Y generic projective curves over $\mathbb{C}$ of degrees $\deg X$ and $\deg Y$, then:

X and Y intersect (with multiplicities) $\deg X \deg Y$ times

- Over $\mathbb{R}$ one gets $\leq$ instead of $=$
- There is a version over any field, and also a higher dimensional version
- Two circles intersect 4 times, which uses $\mathbb{C}$ and ∞, namely $(1 : \pm i : 0)$



Multiplicities



Bézout's theorem for a circle and an ellipse depends on the multiplicities

Thank you for your attention!

I hope that was of some help.