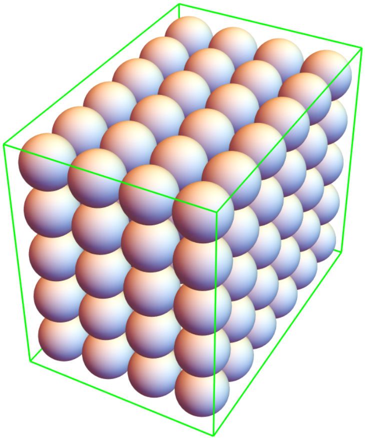
What is...sphere packing?

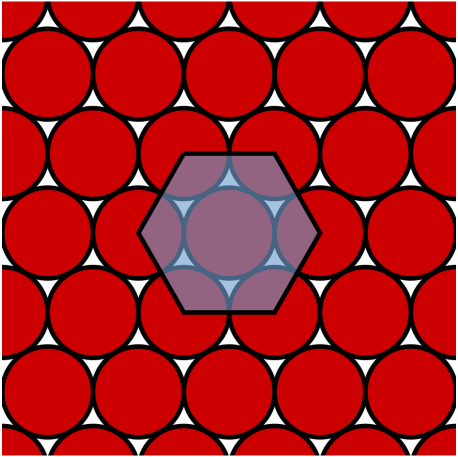
Or: Honeycombs in higher dimensions

Packing spheres



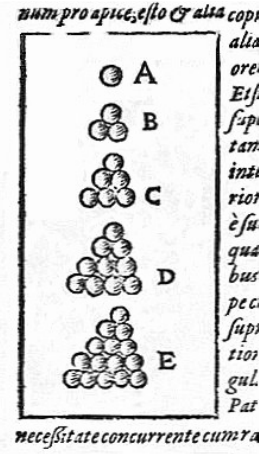
- Sphere packing Arrange unit spheres to fill up the most space
- The name ball packing might be more appropriate (ball=filled sphere)

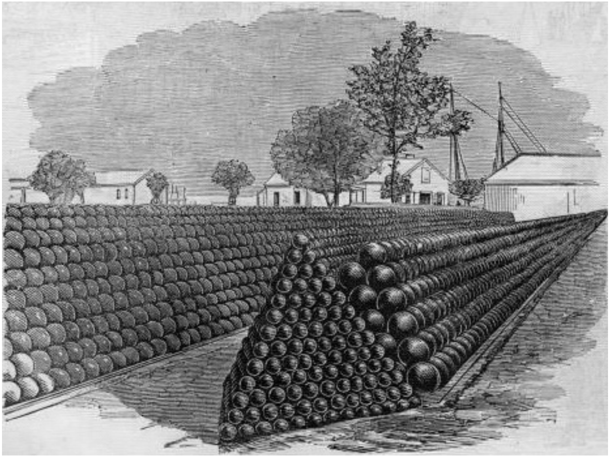
2D



- In 2D the densest packing is hexagonal Bees
- The packing density is about 0.91
- This is relatively easy to prove

3D





- In 3D the densest packing is hexagonal or face-centered Cannonballs
- The packing density is about 0.74
- This is very hard to prove (keyword: Kepler's conjecture)

Enter, the theorem(s)

The optimal packing for spheres is known in...

- ...dimension two Bees
- ...dimension three Cannonballs
- ...some higher dimensions including 8 and 24 E8 and Leech lattice

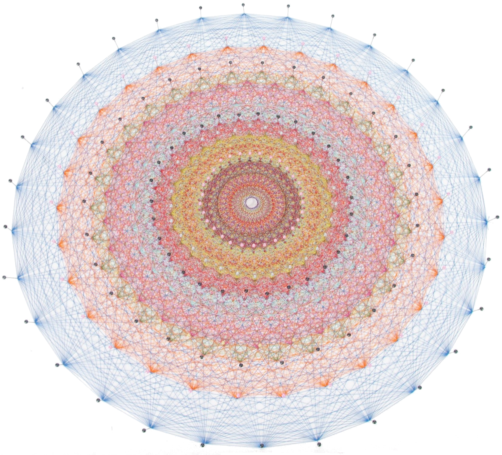
Restricting to lattices makes life much easier:

n	1	2	3	4	5	6	7	8	24
Λ	A_1	A_2	A_3	D_4	D_5	E_6	E_7	E_8	Leech
due to		Lagrange	Gauss	Korkine-Zolotareff		Blichfeldt			Cohn-Kumar

However:

Folk conjecture. For high dimensions the densest packings should be non-lattice

Dimensions 8 and 24



$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 \end{pmatrix}$$

- $\sim$ 2016: The E8 lattice packing is the densest sphere packing in $\mathbb{R}^8$
- $\sim$ 2016: The Leech lattice packing is the densest sphere packing in $\mathbb{R}^{24}$

Thank you for your attention!

I hope that was of some help.