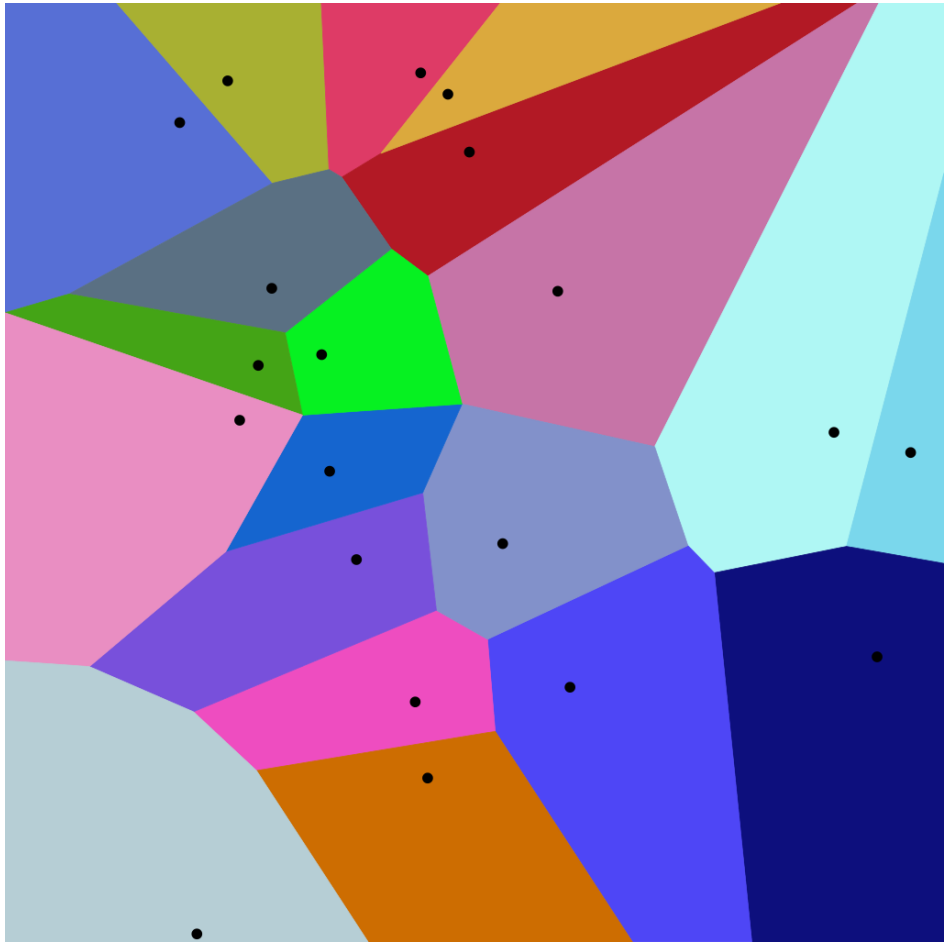
What is...a Voronoi diagram?

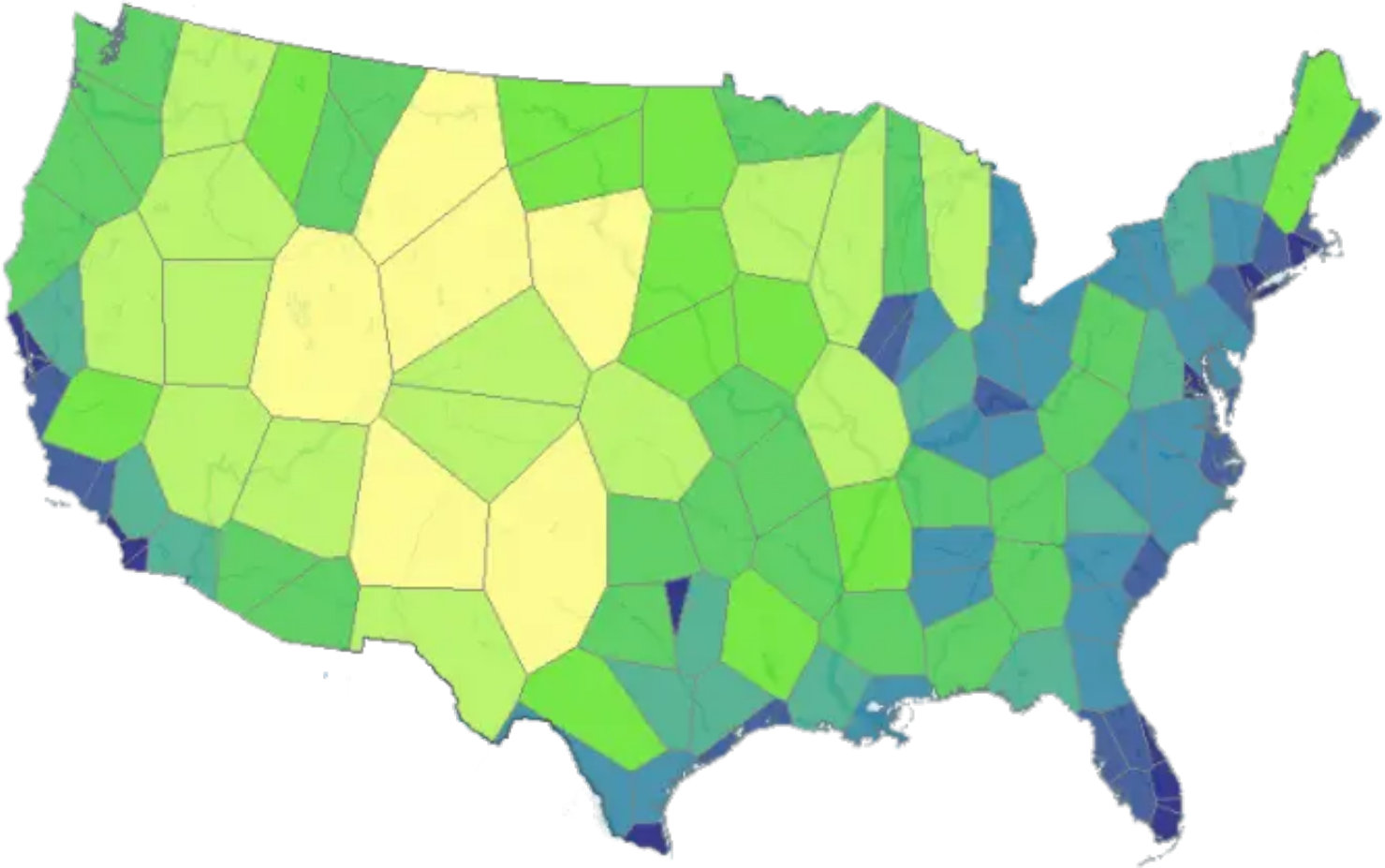
Or: Distance diagrammatically

Voronoi diagrams (VD)



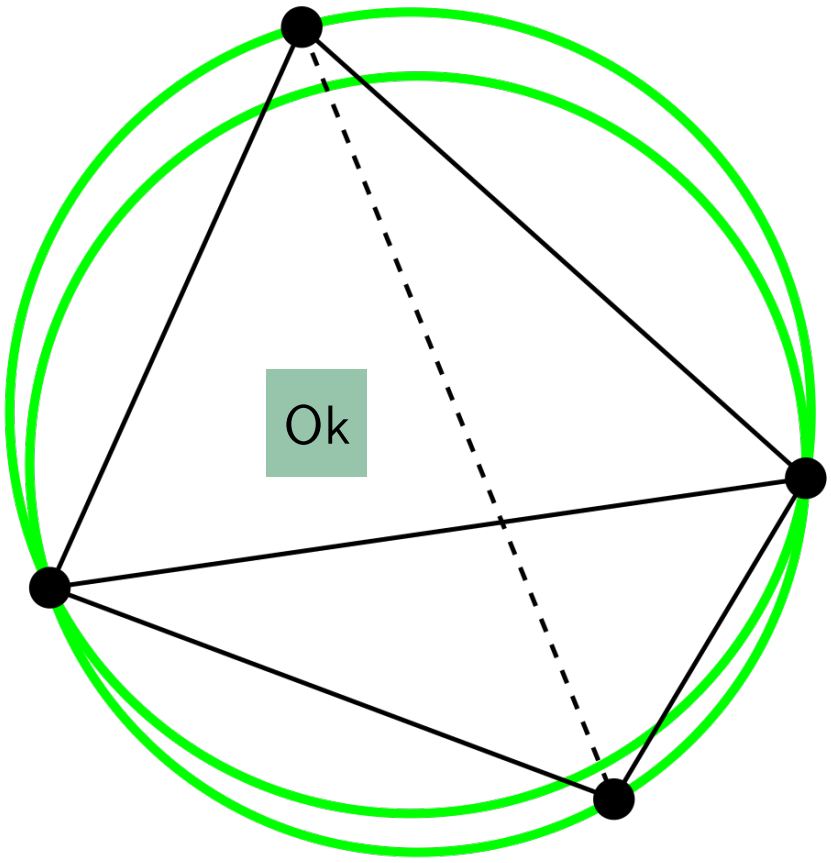
- Fix a number of points in the plane Seeds
- Each seed gets an associated cell
- Cells consist of all points closer to that seed than to any other

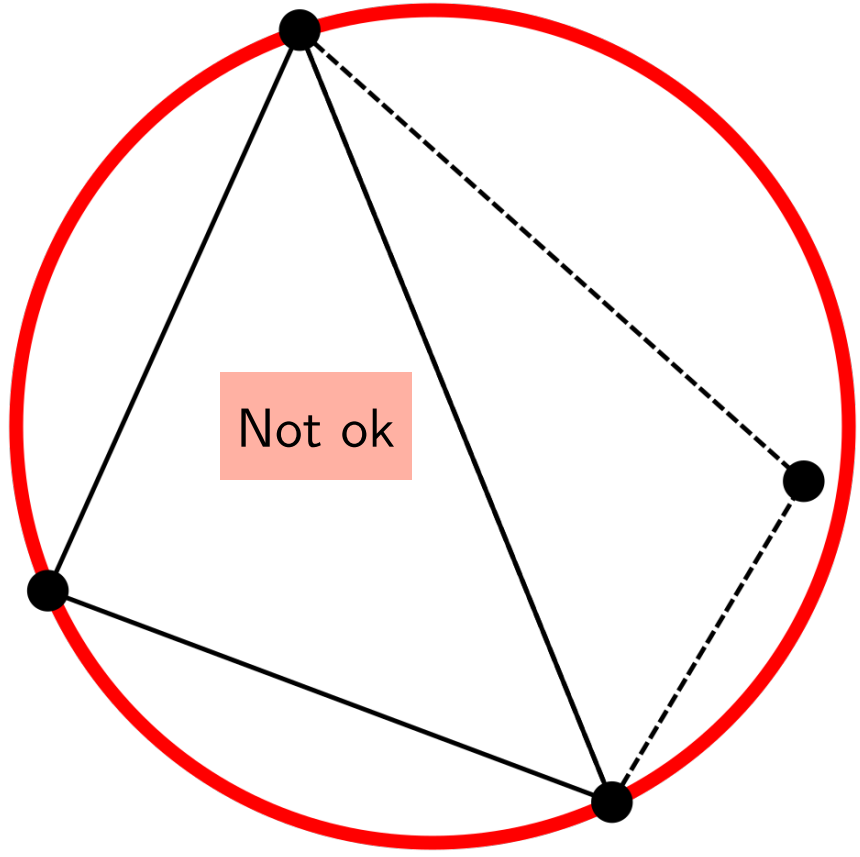
This models real life!



- Above Voronoi cells for airports in the US
- VDs appear in science and real life whenever distance is involved
- There are also "less obvious" applications

Voronoi and triangulations

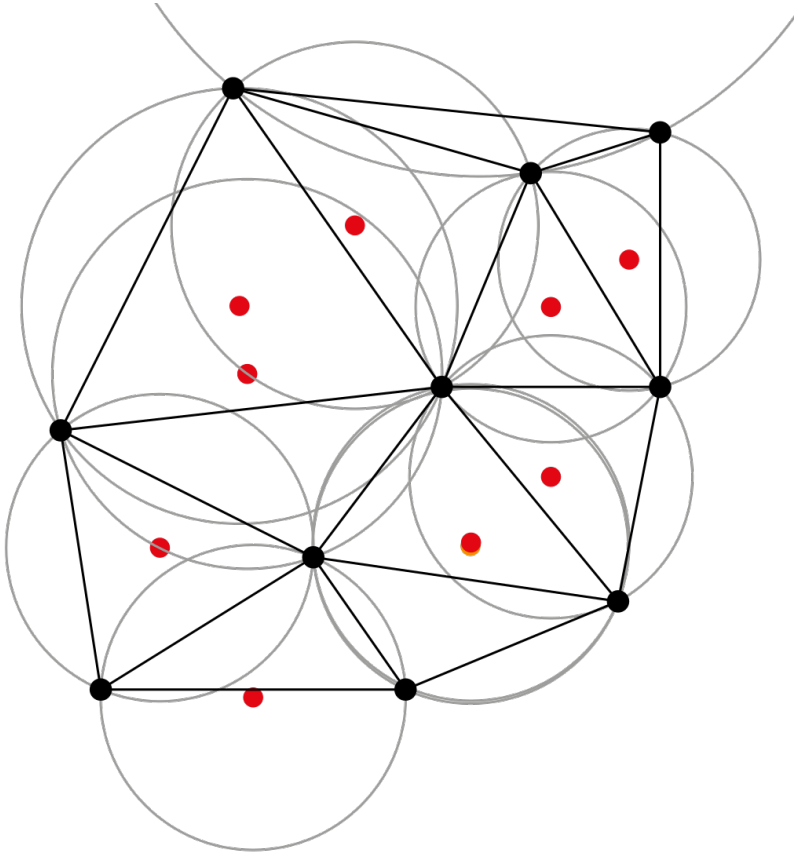


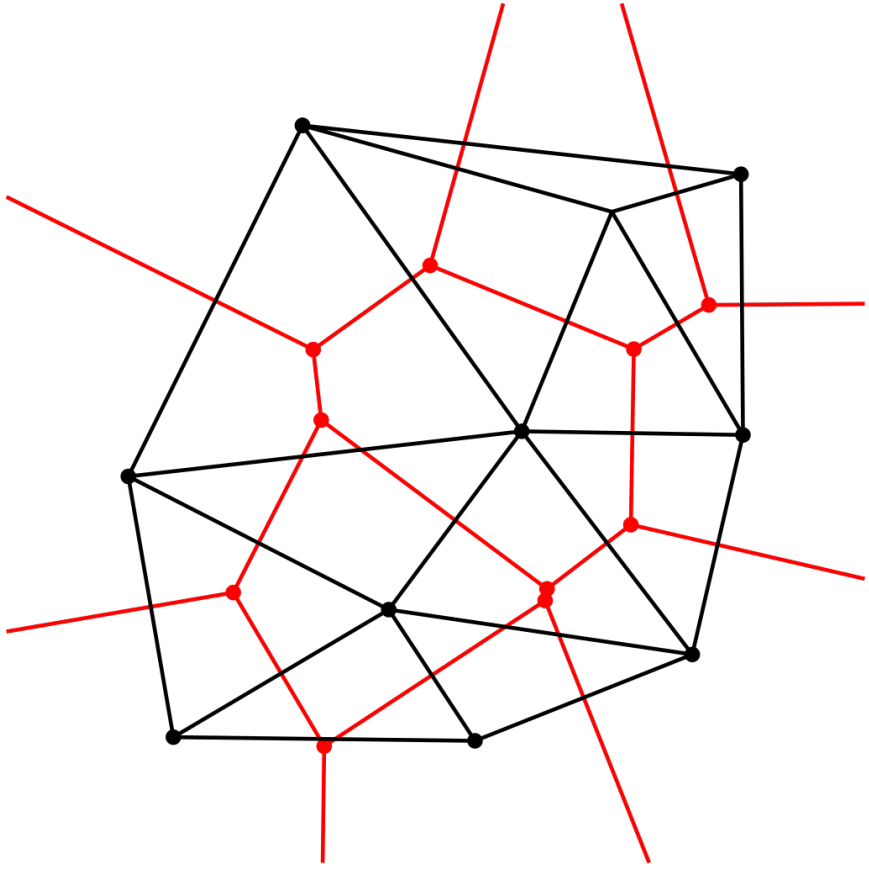


- Fix a number of points in the plane (in general position)
- Triangulate such that no point is inside the circumcircle of any triangle
- Do such Delaunay triangulations (DT) exist?

Enter, the theorem

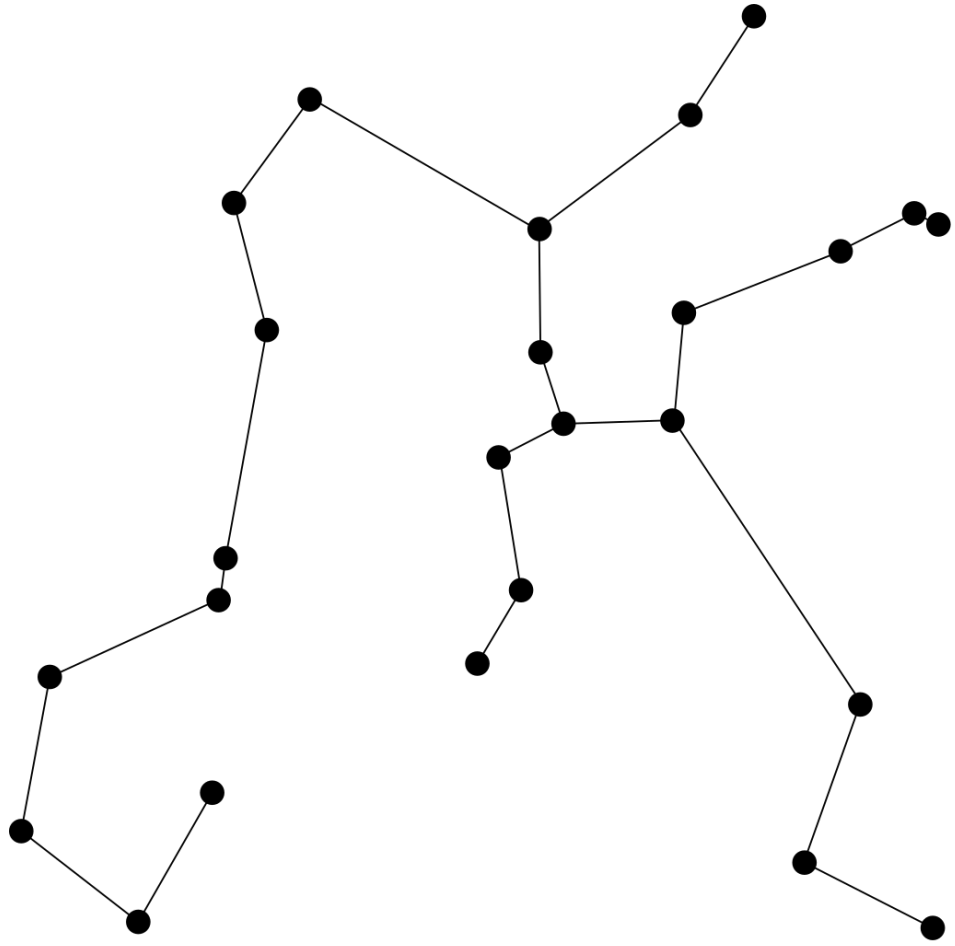
VDs and DTs are dual, so DTs exist





- This duality has many surprising applications One up next!
- Similar constructions work in/with any dimension/metric

A surprising application: Euclidean minimum spanning trees (EMSTs)



- Problem Connect points via line segments that minimize the sum of the distances
- EMST is a subset of DT
- This can be exploited to compute the EMST efficiently

Thank you for your attention!

I hope that was of some help.