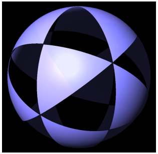
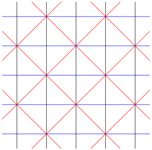
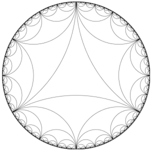
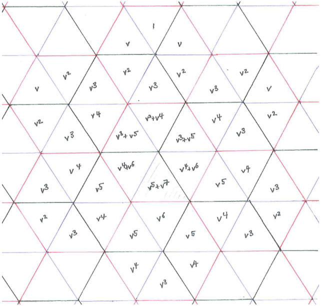
What is...Kazhdan–Lusztig combinatorics?

Or: It is nonnegative!?

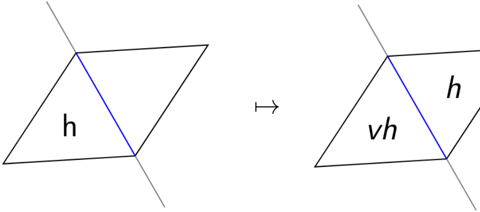
The Kazhdan–Lusztig (KL) goal

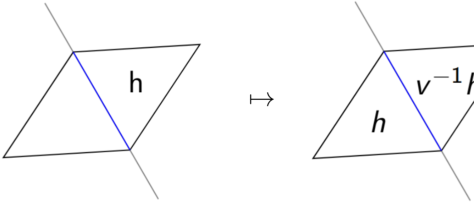



- Associate a pattern to every alcove of a Coxeter complex
- The pattern is indexed by a 1 in a leading alcove
- All other alcoves have an associated polynomial in v and v^{-1}

The local KL rules

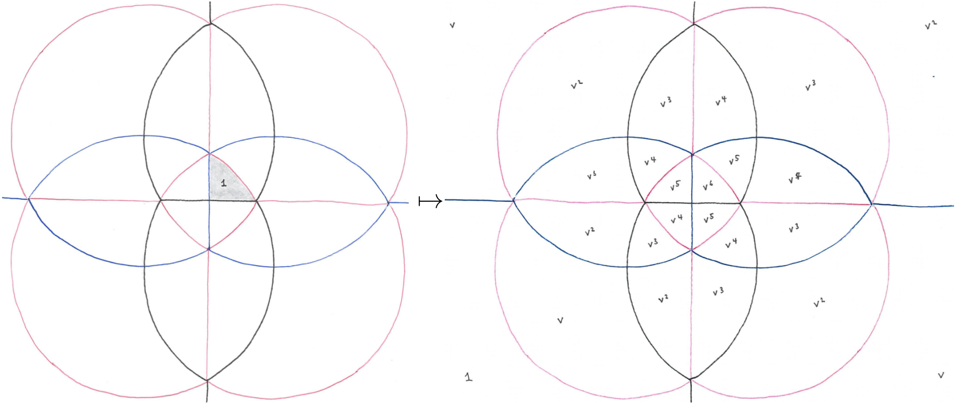


Origin to the southwest



- Up rule If we move away from the origin, then we leave a v behind
- Down rule If we move towards from the origin, then we leave a v^{-1} behind

The KL game



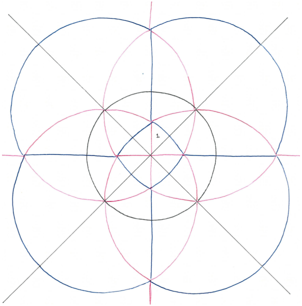
- Start somewhere by putting a 1 Initiation
- Inductively move using the local KL rules applied to the same color
- Whenever you hit a nonleading 1 subtract the corresponding pattern

Enter, the theorem

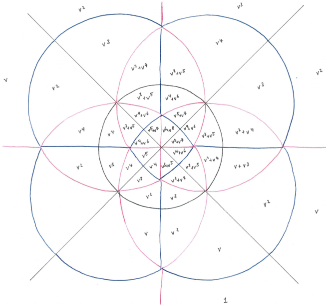
The KL game works and produces entries in $v\mathbb{N}[v]$ (unless leading)

Surprise 1: natural numbers!

Surprise 2: no negative powers!



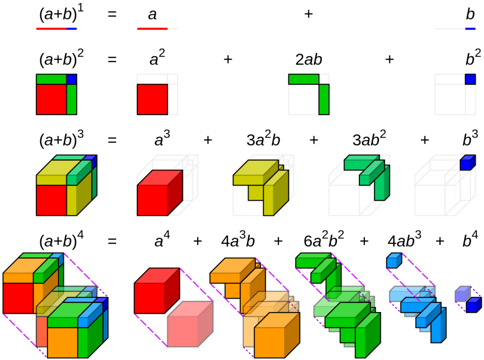
$\mapsto$



- The entries of the alcoves are the KL polynomials (up to conventions)
- **Theorem** "Any" polynomial is a KL polynomial

Natural numbers = counting

Proof that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is a natural number:



- Natural numbers = we count something
- KL polynomials count *e.g.* dimensions of intersection cohomology
- In general KL polynomials count dimensions in Soergel bimodules

Thank you for your attention!

I hope that was of some help.