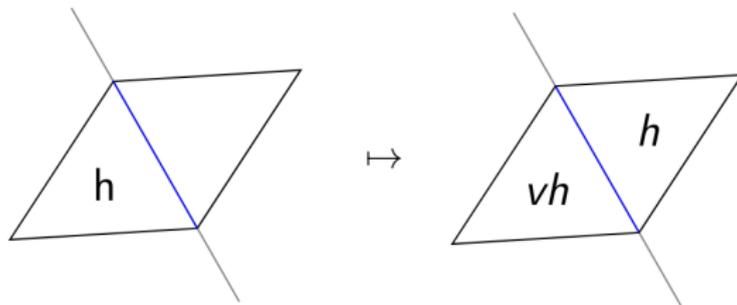


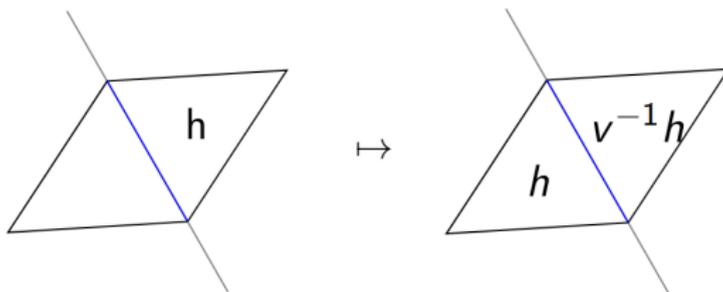
What is...Kazhdan–Lusztig combinatorics?

Or: It is nonnegative!?

The local KL rules

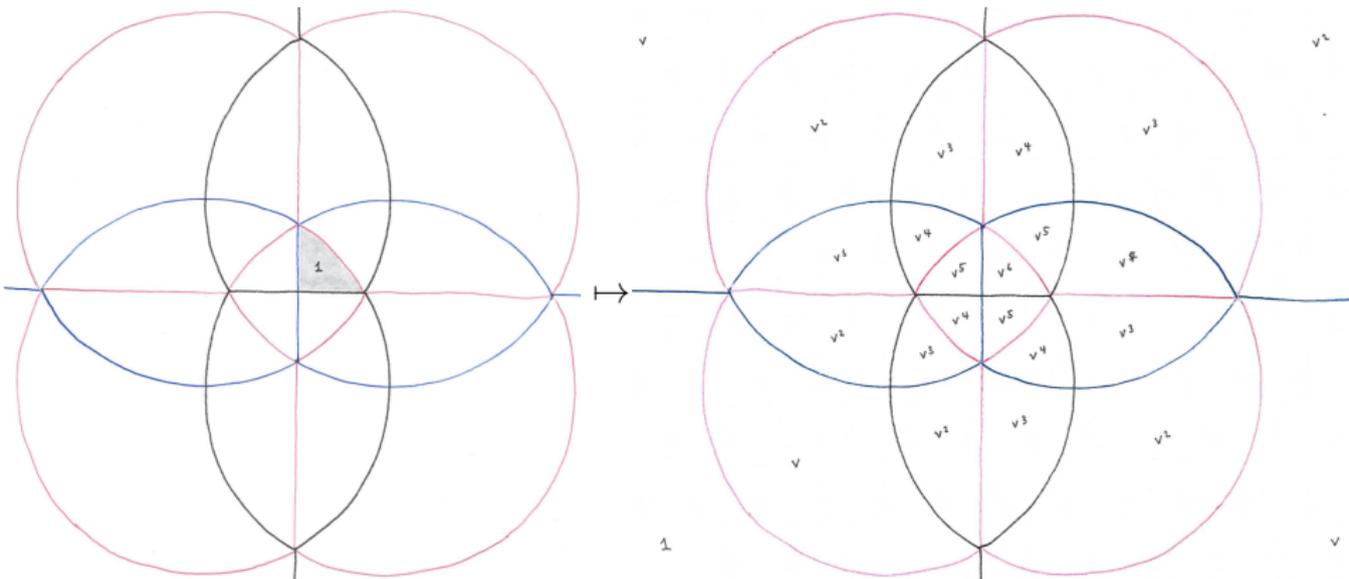


Origin to the southwest



-
- ▶ **Up rule** If we move away from the origin, then we leave a v behind
 - ▶ **Down rule** If we move towards from the origin, then we leave a v^{-1} behind

The KL game



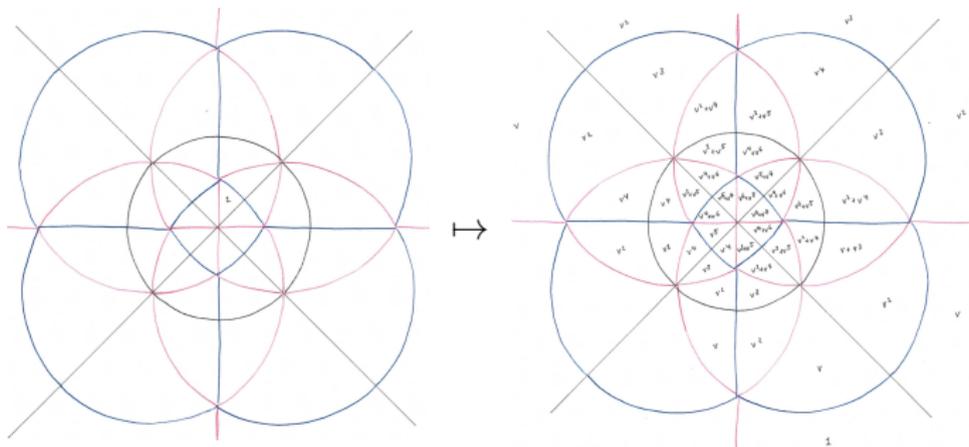
- ▶ Start somewhere by putting a 1 **Initiation**
- ▶ **Inductively move** using the local KL rules applied to the same color
- ▶ Whenever you hit a nonleading 1 **subtract** the corresponding pattern

Enter, the theorem

The KL game works and produces entries in $\nu\mathbb{N}[\nu]$ (unless leading)

Surprise 1: natural numbers!

Surprise 2: no negative powers!



- ▶ The entries of the alcoves are the KL polynomials (up to conventions)
- ▶ **Theorem** “Any” polynomial is a KL polynomial

Natural numbers = counting

Proof that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is a natural number:

$$\begin{aligned}(a+b)^1 &= a + b \\(a+b)^2 &= a^2 + 2ab + b^2 \\(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\end{aligned}$$

The diagram illustrates the expansion of binomial coefficients using colored blocks. For $(a+b)^1$, a red block represents a and a blue block represents b . For $(a+b)^2$, a 2x2 grid of blocks shows a^2 (two red blocks), $2ab$ (two blocks, one red and one blue), and b^2 (two blue blocks). For $(a+b)^3$, a 3x3x3 cube is decomposed into a^3 (red), $3a^2b$ (three yellow blocks), $3ab^2$ (three green blocks), and b^3 (blue). For $(a+b)^4$, a 4x4x4x4 hypercube is decomposed into a^4 (red), $4a^3b$ (four orange blocks), $6a^2b^2$ (six green blocks), $4ab^3$ (four blue blocks), and b^4 (one blue block).

- ▶ Natural numbers = we count something
- ▶ KL polynomials count e.g. dimensions of intersection cohomology
- ▶ In general KL polynomials count dimensions in Soergel bimodules

Thank you for your attention!

I hope that was of some help.