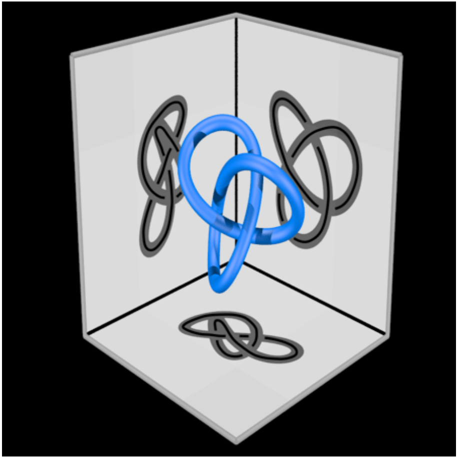
What are...prime knots?

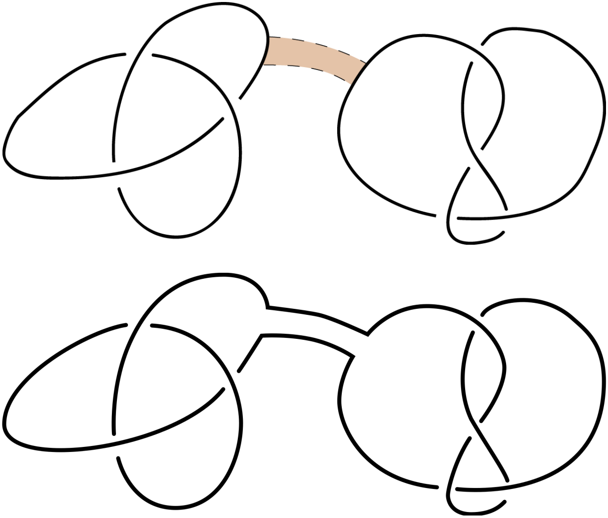
Or: The prime numbers of knot theory!?

Shadows



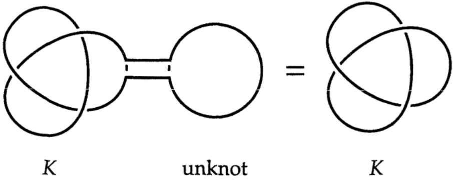
- A knot is a rope with closed ends in three space
- A knot diagram is a projection of a knot (equivalence class)
- What are the elements/prime numbers of knot theory?

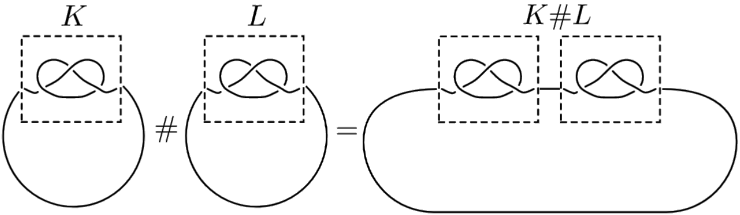
Connected sum $\#$



- The connected sum is an operation much like multiplication
- If "connected sum=multiplication", then what are the prime numbers ?

This is really like multiplication!





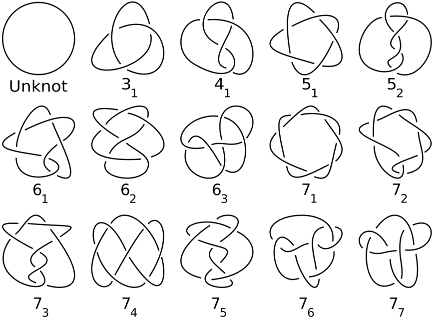
- We have $K\#\text{Unknot} = K$ See above
- We have $K\#L = L\#K$ See above
- We have $K\#(L\#M) = K\#(L\#M)$ Visualization exercise ;-)

Enter, the theorem(s)

A nontrivial knot K is called prime if $K = L\#M$ implies $L = \text{Unknot}$ or $M = \text{Unknot}$

Theorems

- There are infinitely many prime knots; here are a few:

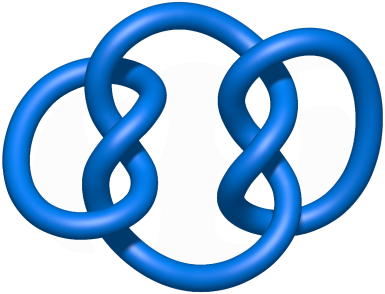


- Every knot can be factored $K = K_1\#...\#K_n$ for prime knots K_i
- This factorization is unique up to permutation of factors

Primality tests? Well...

From mathworld:

In general, it is nontrivial to determine if a given knot is prime or composite (Hoste *et al.* 1998). However, in the case of alternating knots, Menasco (1984) showed that a reduced alternating diagram represents a prime knot iff the diagram is itself prime ("an alternating knot is prime iff it looks prime"; Hoste *et al.* 1998).



- Detecting primality is hard – for knots and numbers
- Sometimes this is easy, *e.g.* the above knot is composite Visualization exercise ;-)

Thank you for your attention!

I hope that was of some help.