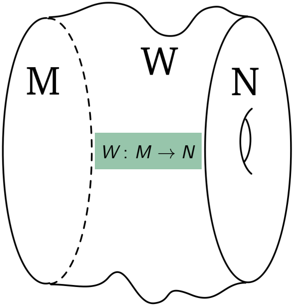
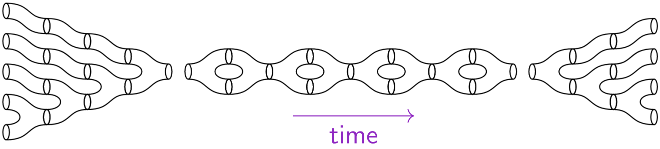
What are...diagram algebras?

Or: Pictures in algebra and algebra in pictures

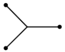
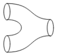
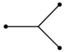
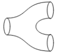
Cobordisms

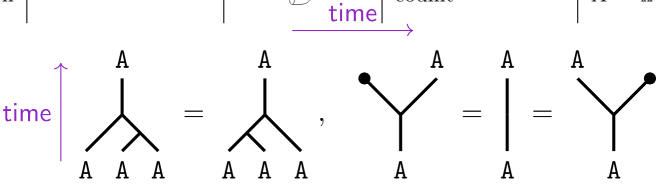




- Slogan A cobordism is a map between manifolds
- Gluing defines a natural multiplication on cobordisms
- Idea Use this as a guide to construct nice algebraic objects!

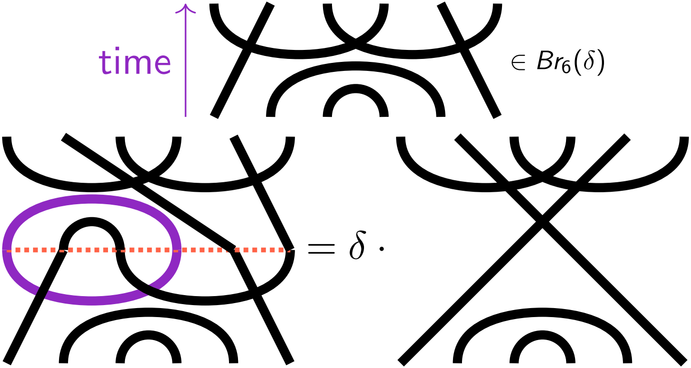
Cobordisms = some form of algebra

Principle	Feynman diagram	2D cobordism	Algebraic operation (in a $\Bbbk$-algebra A)	
merging			multiplication	$A \otimes A \to A$
creation			unit	$\Bbbk \to A$
splitting			comultiplication	$A \to A \otimes A$
annihilation			counit	$A \to \Bbbk$



- In some sense, cobordisms are modeling algebras Let us explore this!
- Beware: there are different reading conventions in the literature (and in this video – my apologies)

Brauer diagrams



- Brauer algebra = 1d cobordisms diagrams from n to n points without circles
- Multiplication is stacking and circle removal

$$\bigcirc \mapsto \delta \cdot \emptyset$$

- The Brauer algebra originates in invariant theory

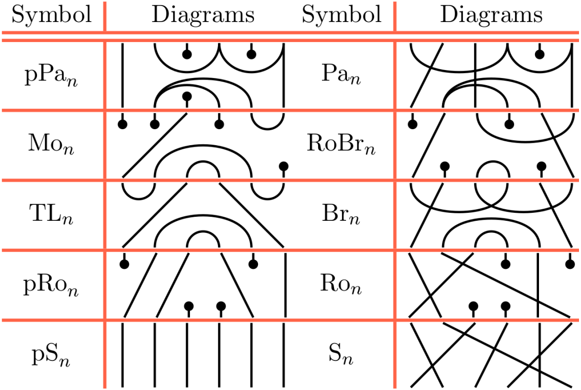
Enter, the theorem

Slogan Diagram algebras = algebras modeled on cobordisms

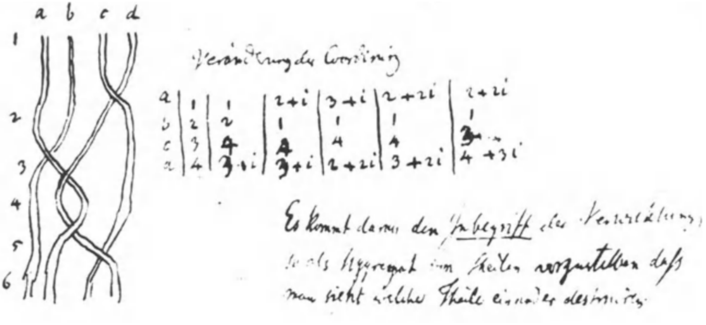
Theorem Diagram algebras have an associative multiplication

Proof? Use cobordism picture + a bit of extra work

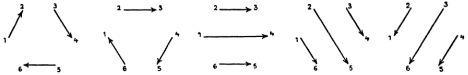
Some other diagram algebras:



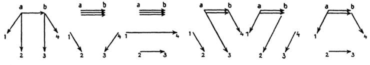
Connecting different fields



3. Als zweites Beispiel behandeln wir die zyklische einvalentige Kette mit sechs Atomen. Die Valenzfunktionen sind

(4)


Als Beispiel betrachten wir das Hydrazin NH_2—NH_2. Wir bezeichnen mit a, b die beiden N-Atome, mit 1, 2, 3, 4 die vier H-Atome. Ordnen wir die Atome auf einem Kreis an, so erhalten wir nach der Anweisung folgende sechs Valenzzustände als Basis[3]):

(3)


- Top Gauss' handwritten notes on braids diagrams and magnetism ∼1800++
- Bottom Rumer–Teller–Weyl: diagram algebras and chemical bonding ∼1932

Thank you for your attention!

I hope that was of some help.