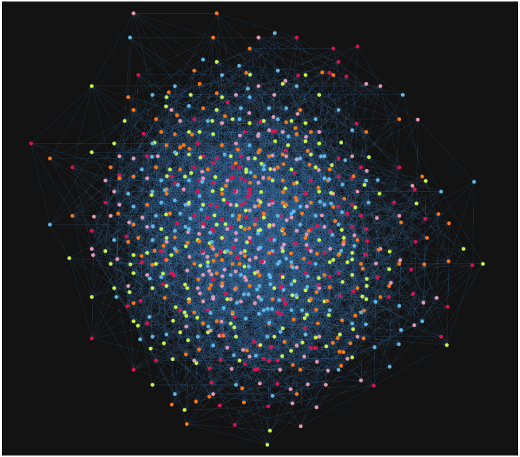
What is...a pointwise coloring of the plane?

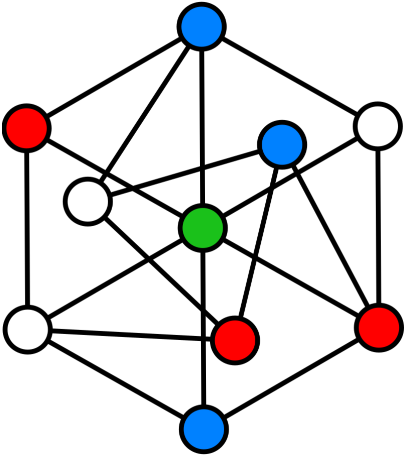
Or: The chromatic number of the plane

Four colors suffice



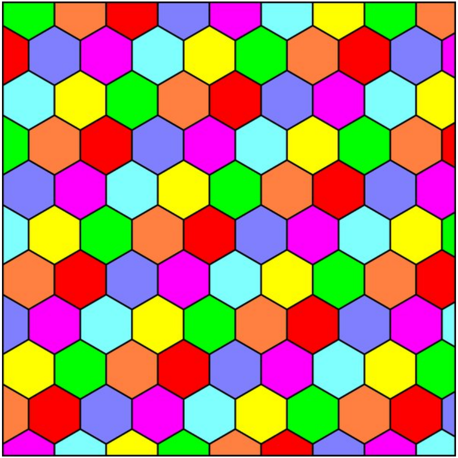
- We want to assign a color to each point in the plane E^2...
- ...such that no two points of the same color are unit distance apart
- Problem Find the minimal number $\chi(E^2)$ of colors

A lower bound



- Idea Phrase this problem in terms of colorings for unit distance graphs
- Above a unit distance graph that needs four colors
- Hence, we get the lower bound $4 \leq \chi(E^2)$

An upper bound

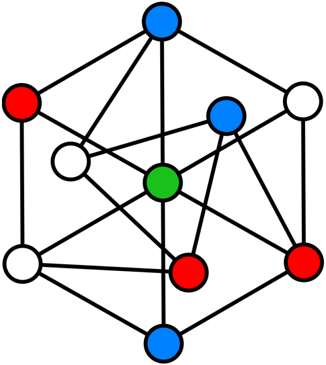
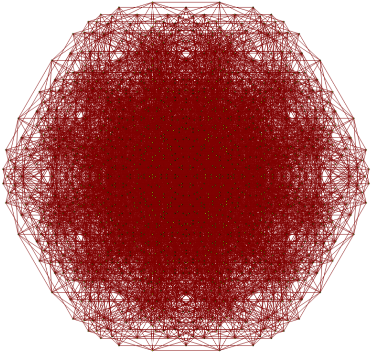


- The hexagon tessellation is seven colorable
- Using diameter slightly less than one gives a coloring of the plane
- Hence, we get the upper bound $\chi(E^2) \leq 7$

Enter, the theorem(s)

$4 \leq \chi(E^2) \leq 7$

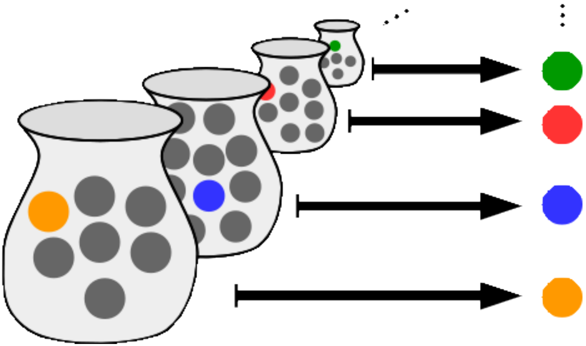
- Actually $5 \leq \chi(E^2) \leq 7$: in 2018 a >1000-vertex unit distance graph was found with a computer

4:  , 5: 

- There are unit distance graph with chromatic number 5 and ≤509 vertices

Polymath project

Enter, the axioms of set theory



- Construct the unit distance graph G with vertices $= E^2$ and edges

$$(a \leftrightarrow b) \Leftrightarrow (a - b \in \mathbb{Q}^2, |a - b| = 1)$$

- Theorem (rough version) If the axiom of choice holds, then $\chi(G) = 2$ and otherwise $3 \leq \chi(G) \leq 7$
- Something similar might be true for $\chi(E^2)$ itself (but this became unlikely with the lower bound 5)

Thank you for your attention!

I hope that was of some help.