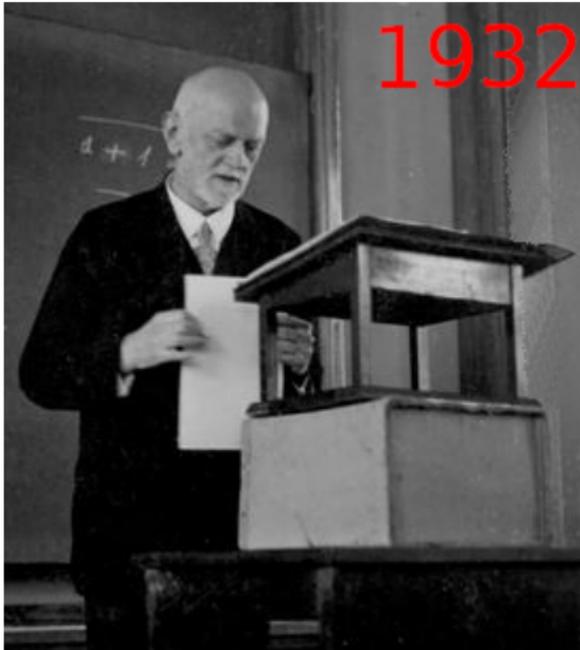


What is...P versus NP?

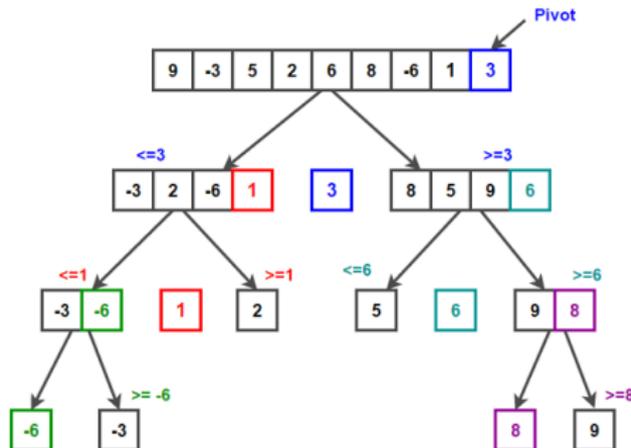
Or: Building a chair versus recognizing it

Hilbert's 1900 speech



- ▶ Hilbert Is there a “purely mechanical procedure” to “check statements”?
- ▶ The $P \stackrel{?}{=} NP$ problem is a modern refinement of Hilbert's question
- ▶ $P = NP$ \leftrightarrow fast way to address all questions with mechanically verifiable answers

Jigsaw versus sorting



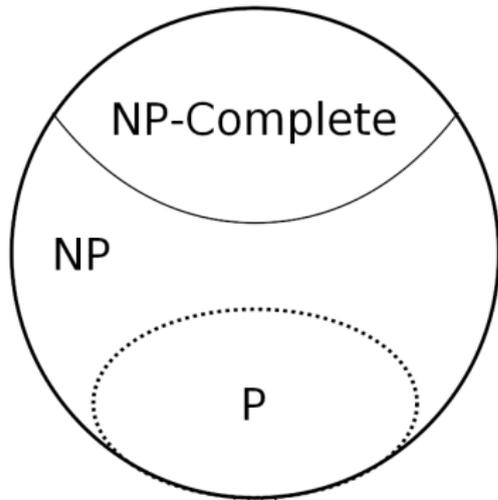
NP = easy to check that one has a solution & brute-force gives solutions + technicalities

P = easy to find a solution, $P \subset NP$

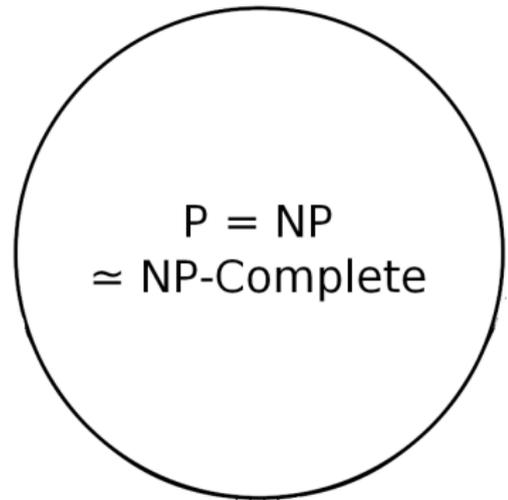
$P=NP$ "=" anyone who can recognize a chair can also build one

- ▶ It is not necessarily easy to find a solution to a jigsaw puzzle but it is easy to check whether one has a solution **Jigsaw in NP**
- ▶ Sorting can be done $\leq n^2$, and thus it is easy to find a solution **Sorting in P**

NP-complete



$P \neq NP$



$P = NP$

- ▶ NP-complete = problems that can be used to simulate every NP problem
- ▶ $(P \cap \text{NP-complete} \neq \emptyset) \Leftrightarrow (P = NP)$
- ▶ **Task** Find NP-complete problems

Enter, the theorem(s)

3SAT is NP-complete, and 3SAT can be solved in $O(1.3^n)$

- ▶ SAT: the problem of determining if there exists an interpretation that satisfies a given Boolean formula; 3SAT = SAT with three inputs
- ▶ Most known NP-complete problems are reduced to some form of SAT

Rules of the game

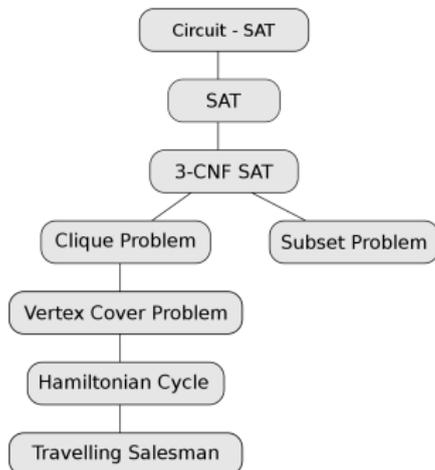
The game is won when at least one cell on each line is green. Clicking on a number will color each cell with the same number in green, and each cell with the opposite number in red. Clicking on a colored cell will remove both colors.

👉 points out lines which don't yet contain a green cell

▶ points out lines where there's only one chance left to put a green cell

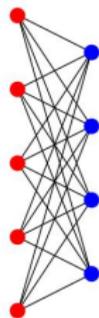
🚫 points out lines which are completely red

-3	-8	-5	👉
3	1	7	👉
-1	2	9	👉
-6	-3	-9	👉
-9	-3	-4	👉
2	9	-5	👉
-8	5	-7	👉
2	-9	-6	👉
-7	-4	-1	👉
9	-6	3	👉

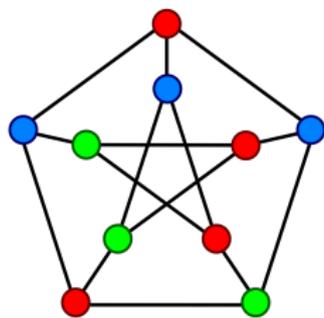


Why is $P \neq NP$ so hard to prove?

2-colorability
 $\in P$



3-colorability
 $\in NP$ -complete



All LP problems
are $\in P$



Matrix mult.
is in $O(n^{2.4})$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} u & v \\ w & x \\ y & z \end{bmatrix}$$
$$= \begin{bmatrix} au + bw + cy & av + bx + cz \\ du + ew + fy & dv + ex + fz \end{bmatrix}$$

- ▶ 1) Variations of problems differ a lot in their complexity
- ▶ 2) There are amazingly clever ways to avoid brute-force approaches
- ▶ 3) Complexity classes are hard to determine precisely

Thank you for your attention!

I hope that was of some help.