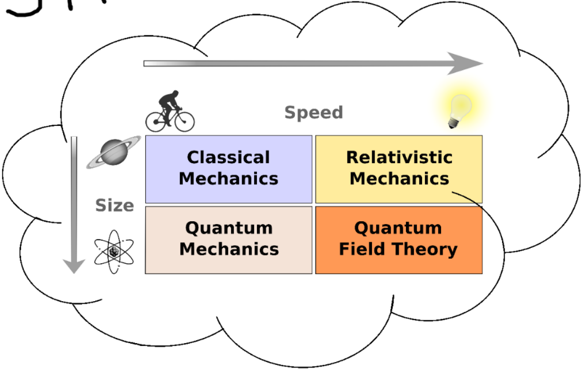
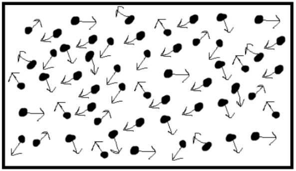
What is...the square ice constant?

Or: Ice and 1.539601...

STATISTICAL MECHANICS



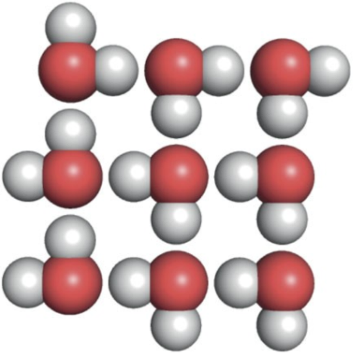
Box of Gas

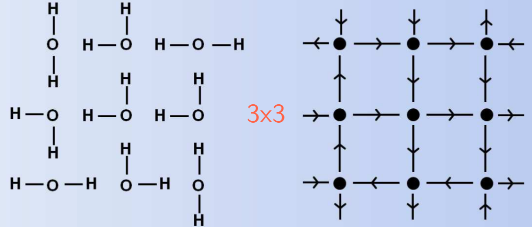


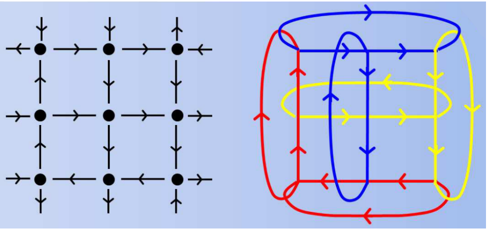
Macrostates instead of microstates

- Statistical mechanics is a branch of physics that pervades all other branches
- Very often physical systems are modeled
- Experience tells us that real world models $\Rightarrow$ nice mathematics

Ice modeled (we ignore whether the model makes sense physically...)

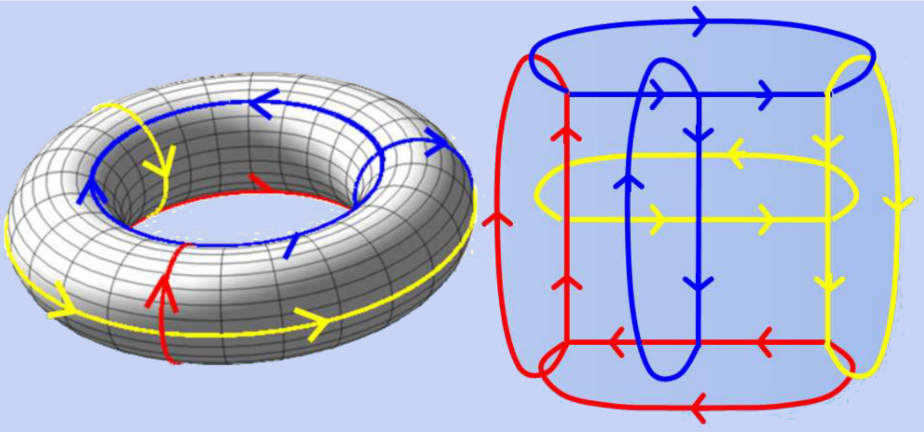






- Ice forms a crystal of which we think a living on an nxn square lattice
- Orient the lattice according to the bonding
- We get an orientation for a square graph

Consider the limit



- In order to avoid boundary nonsense we think of this as living on a torus
- Eulerian orientation = each vertex has two incoming and two outgoing edges
- Goal Count the number of Eulerian orientations on an nxn square for $n \to \infty$
- Note that Eulerian orientations are the ones that make sense physically

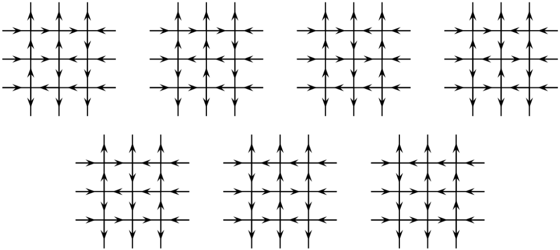
Enter, the theorem

The number of Eulerian orientations f_n satisfies

$$\lim_{n\to\infty} \sqrt[2n]{f_n} = \frac{8\sqrt{3}}{9} \approx 1.539601...$$

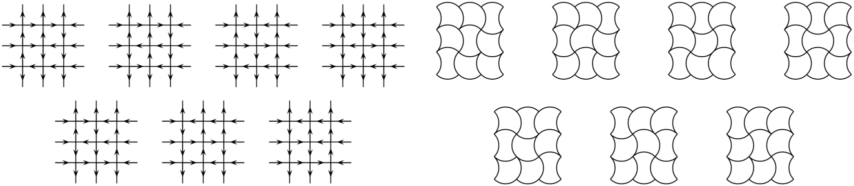
- The number f_n itself approaches ∞

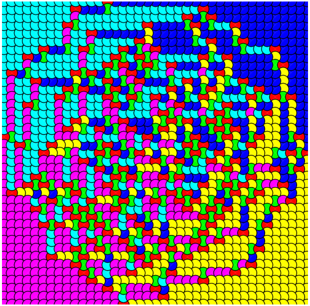
$n = 3$ has $f_n = 7$:



- 1.539601... = Lieb's square ice constant
- This relates to the residual entropy of square ice via the six vertex model
- Counting f_n for other lattices (like other ice lattices) is very difficult

Tilings and ice





- The six local states correspond to six tilting patterns
- This was used to give an ice-model-proof of the alternating-sign matrix conjecture

Thank you for your attention!

I hope that was of some help.