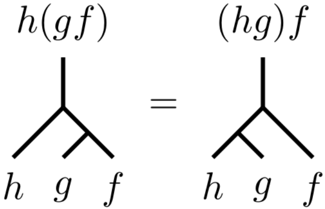
What is...quantum topology - part 14?

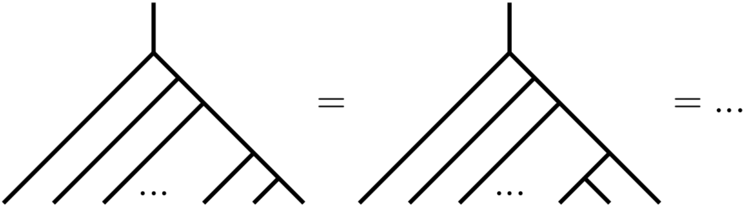
Or: Monoidal categories 2 from Chapter 2

A “wrong” and a “correct” definition

“Wrong” :

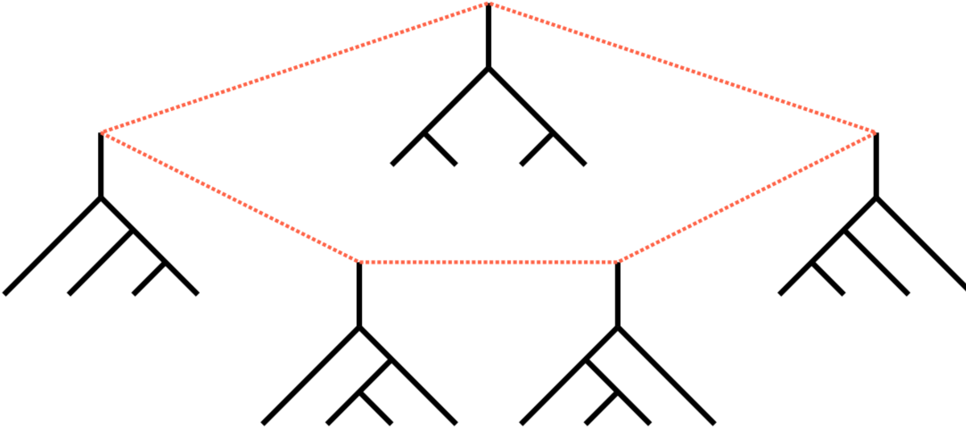


“Correct” :



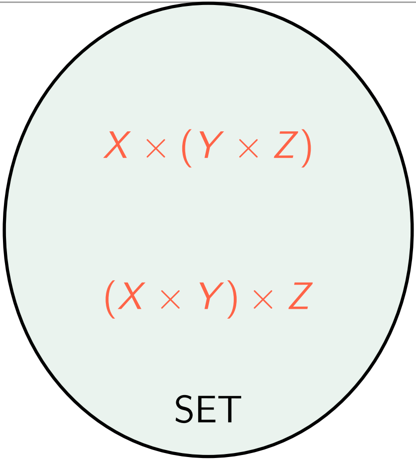
- (A) $h \cdot (g \cdot f) = (h \cdot g) \cdot f$
- (B) Same result regardless of how valid pairs of parentheses are inserted
- “Philosophically correct” Use (B) as the definition and show that (A) ⇔ (B)

Strategical interlude



- Define a monoid/group/... using $h \cdot (g \cdot f) = (h \cdot g) \cdot f$
- Show that $h \cdot (g \cdot f) = (h \cdot g) \cdot f$ implies all bracketings Coherence theorem
- Forget parenthesis altogether Strictification

Why parenthesis in the first place?



- $X \times (Y \times Z) \neq (X \times Y) \times Z$ as sets Set theory is inflexible
- In order to make SET with $\otimes = \times$ monoidal we need parenthesis
- Use an equivalent category and avoid parenthesis Category theory is flexible

For completeness: A formal definition

A strict monoidal category $(C, \otimes, \mathbb{1})$ consists of

- A category C
- A bifunctor $\otimes\colon C \times C \to C$ (write $XY = X \otimes Y$)
- A unit object $\mathbb{1} \in C$

such that

(a) *associativity* holds, *i.e.*

$$X(YZ) = (XY)Z, \quad h \otimes (g \otimes f) = (h \otimes g) \otimes f$$

(b) the *identity law* holds, *i.e.*

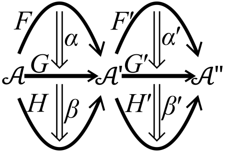
$$\mathbb{1}X = X = X\mathbb{1}, \quad \mathrm{id} \otimes f = f = f \otimes \mathrm{id}$$

Theorem (strictification)

Every monoidal category is monoidally equivalent to a strict monoidal category

Some examples

Name	$\otimes$	Strict?	Strictification
Set	$\times$	No	S(Set)
Cat	$\times$	No	S(Cat)
1Cob	Juxtaposition	Yes	1Cob
nCob	Juxtaposition	Yes	nCob
$\text{Vec}_{\mathbb{K}}$	$\otimes$	No	$S(\text{Vec}_{\mathbb{K}})$
$\text{Vec}_{\mathbb{K}}$	$\oplus$	No	$S(\text{Vec}_{\mathbb{K}})$
End(C)	$\circ$	Yes	End(C)



- Often the skeleton $S(C)$ is the strictification but not always
- In general the strictification is an endofunctor category

Thank you for your attention!

I hope that was of some help.