

**What is...quantum topology - part 17?**

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Or: Monoidal categories 5 from Chapter 3

# Generators-relations

## Groups encoded efficiently

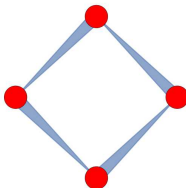
$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\mathbb{Z}/4\mathbb{Z}$  (written additively):

1 is a generator of  $\mathbb{Z}/4\mathbb{Z}$ :

$$\emptyset = 0, \quad 1 = 1, \quad 11 = 1 + 1 = 2, \quad 111 = 1 + 1 + 1 = 3$$

Illustrated as a graph:



- Above A group given as a multiplication table
- Above 2 A group given by generators-relations
- Game we will play Try to find generators-relations of 'nice' categories

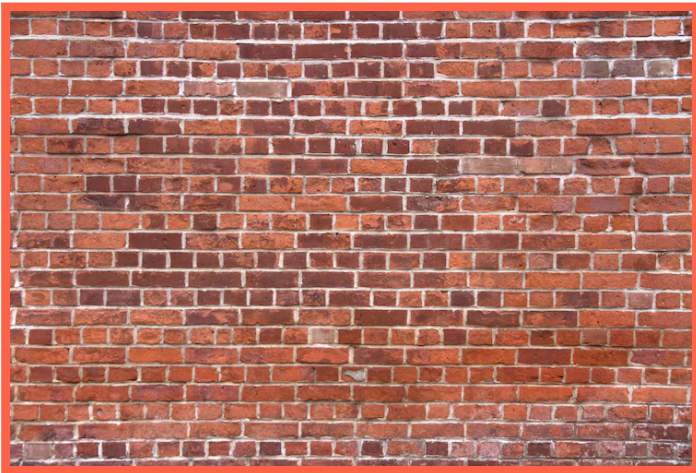
## Generators-relations for sets



- ▶ **Alphabets  $S$**  = the symbols we use (“pearls”); we do not include  $\emptyset$  (id)
- ▶ **Words** = concatenations of pearls on a line
- ▶ **Relations  $R$**  = the ways to manipulate pearls

# Generators-relations for monoidal categories

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- **Objects** As for sets
- **Morphisms** Similarly, but we stack them like bricks; generators  $T$
- **Relations  $R$**  = the ways to manipulate bricks

# For completeness: A formal definition

Annoying but not difficult to write down...

make me a list of the ten most annoying things

Sure — here's a list of ten widely considered annoying things, based on common human experiences. You might relate to a few of them:

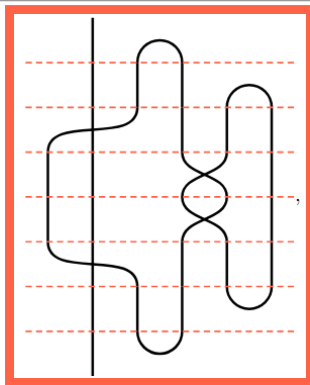
1. **Loud chewing or slurping** – Especially in quiet settings.
2. **Slow walkers in crowded places** – Bonus irritation if they block the whole path.
3. **People who interrupt constantly** – Especially mid-thought.
4. **Unskippable ads (especially back-to-back ones)** – Just let us watch the video!
5. **Getting a song stuck in your head** – And only knowing one line of it.
6. **When autocorrect changes a perfectly fine word** – You meant what you typed!
7. **Wi-Fi cutting out during important moments** – Bonus rage if it's during a live call.
8. **People who leave dishes near the sink** – But not in it or, better, the dishwasher.
9. **"We need to talk" texts with no follow-up** – Instant stress activation.
10. **Getting ghosted after making plans** – Time is precious, don't waste it!

Want a version that's more niche or targeted (like workplace, roommates, or internet behavior)?

Example The Brauer category  $\text{Br}$

$$\begin{aligned} S : \bullet, \quad T : \text{X} : \bullet^2 \rightarrow \bullet^2, \quad \cap : \bullet^2 \rightarrow \mathbb{1}, \quad \cup : \mathbb{1} \rightarrow \bullet^2, \\ R : \left\{ \begin{array}{l} \text{X} = | \quad |, \quad \text{X} = \text{X}, \quad \cup = | = \cup \\ \cap = \cap, \quad \cap = \cup, \quad \cap = \cap \text{X}, \quad \cup = \cup \text{X} \end{array} \right. \end{aligned}$$

## A generators-relations presentation



- Theorem  $\text{Br}$  is monoidally equivalent to  $1\text{Cob}$
- Here is the equivalence :

$$\text{Br} \rightarrow 1\text{Cob}, \bullet \mapsto \bullet, \times \mapsto \times, \cap \mapsto \cap, \cup \mapsto \cup.$$

- Proof? E.g. writing cobordisms in Morse form (as above) shows fullness

**Thank you for your attention!**

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I hope that was of some help.