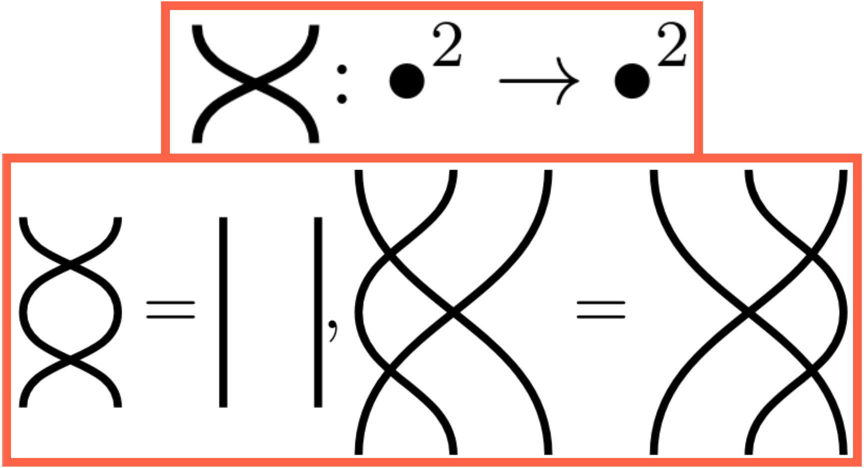
What is...quantum topology - part 21?

Or: Pivotal categories 3 from Chapter 4

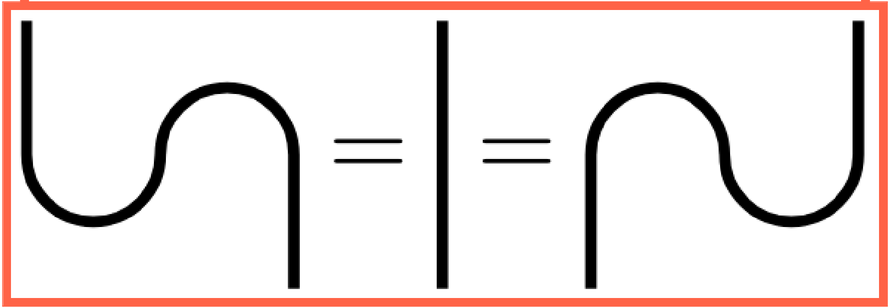
Swapping factors



- Say we have some set $\bullet$; consider $\bullet^d = \bullet \times \ldots \times \bullet$
- Take the swap map: $\bullet^2 \to \bullet^2, (x, y) \mapsto (y, x)$
- Above The diagrammatic algebra of the swap map which investigate soon!

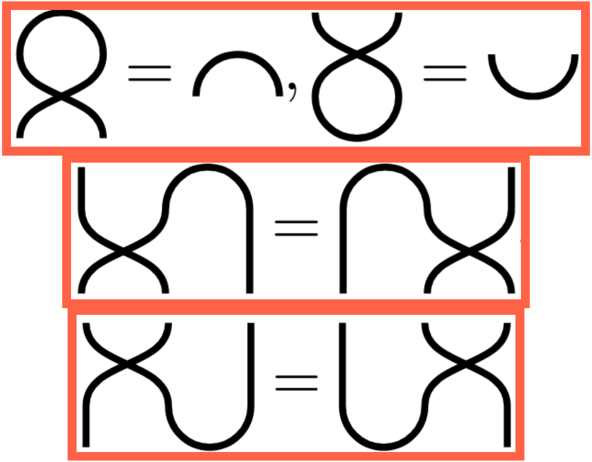
Pairing factors

$$\cap : \bullet^2 \to \mathbb{1}, \cup : \mathbb{1} \to \bullet^2$$



- Say we have some $\mathbb{C} = \mathbb{1}$ vector space $\bullet$; consider $\bullet^d = \bullet \otimes \ldots \otimes \bullet$
- Take a (nondegenerate symmetric) pairing map: $\bullet^2 \to \mathbb{1}, x \otimes y \mapsto \langle x, y \rangle$ (identify $\bullet$ with its dual); take the copairing $\mathbb{1} \to \bullet^2, 1 \mapsto \sum_{basis} x \otimes x^*$
- Above The diagrammatic algebra of the (co)pairing maps (rigid/pivotal)

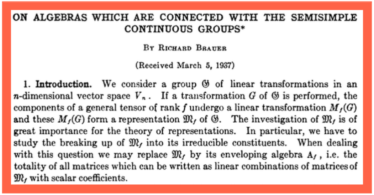
Swapping and pairing



- Say we have some $\mathbb{C} = \mathbb{1}$ vector space $\bullet$; consider $\bullet^d = \bullet \otimes \dots \otimes \bullet$
- Take swapping and pairing together
- Above The diagrammatic algebra of the swapping and (co)pairing maps Br

For completeness: A formal statement

Br gives diagrammatics of orthogonal $O_n(\mathbb{C})$ or symplectic $SP_n(\mathbb{C})$ invariants



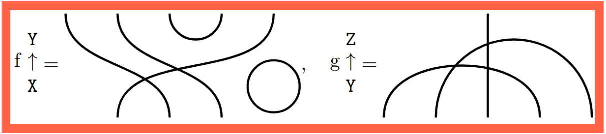
ON ALGEBRAS WHICH ARE CONNECTED WITH THE SEMISIMPLE CONTINUOUS GROUPS*

BY RICHARD BRAUER

(Received March 5, 1937)

1. **Introduction.** We consider a group $\mathfrak{G}$ of linear transformations in an n-dimensional vector space V_n. If a transformation G of $\mathfrak{G}$ is performed, the components of a general tensor of rank f undergo a linear transformation $M_f(G)$ and these $M_f(G)$ form a representation $\mathfrak{M}_f$ of $\mathfrak{G}$. The investigation of $\mathfrak{M}_f$ is of great importance for the theory of representations. In particular, we have to study the breaking up of $\mathfrak{M}_f$ into its irreducible constituents. When dealing with this question we may replace $\mathfrak{M}_f$ by its enveloping algebra A_f, i.e. the totality of all matrices which can be written as linear combinations of matrices of $\mathfrak{M}_f$ with scalar coefficients.

- Formally There is a "full and essentially surjective" functor **Br** $\to$ **Rep**(O) or **Rep**(SP), and the "kernel" can also be described
- Comment $GL_n(\mathbb{C})$ does not have a natural pairing, hence $O_n(\mathbb{C})$, $SP_n(\mathbb{C})$
- We have seen this in more generality (up to the swap)!



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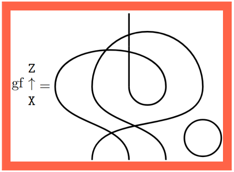
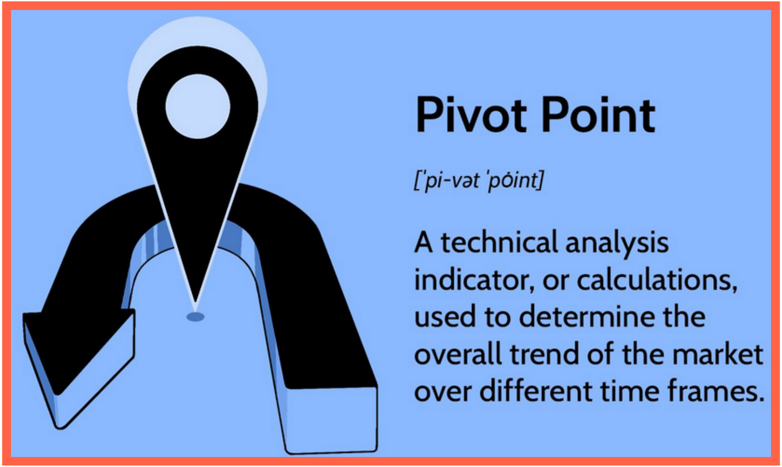


Rotating diagrams



- Diagrammatic algebra seems to originate in 19th century invariant theory
- Pivotal comes from pivot = 'rotation point'
- Rigid is an older name and I do not know where it comes from

Thank you for your attention!

I hope that was of some help.