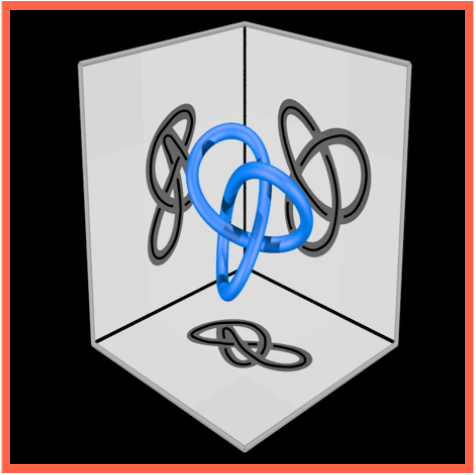
What is...quantum topology - part 22?

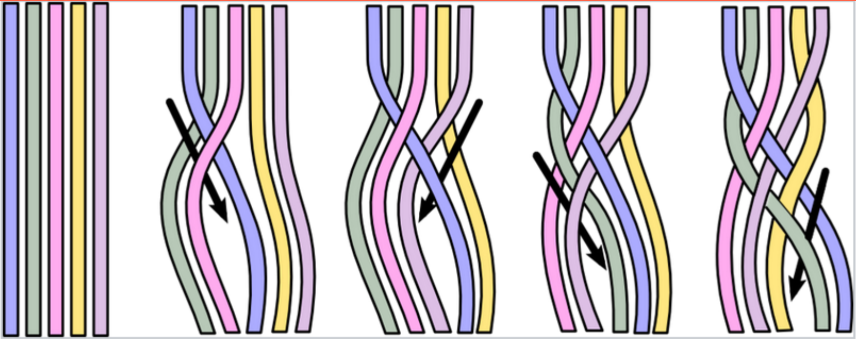
Or: Braided categories 1 from Chapter 5

Braiding: the bird's-eye idea



- Braiding is the categorical avatar of a crossing of strands
- It upgrades "swap" into coherent structure on tensor products
- It is the gateway from monoidal categories to knot-type invariants

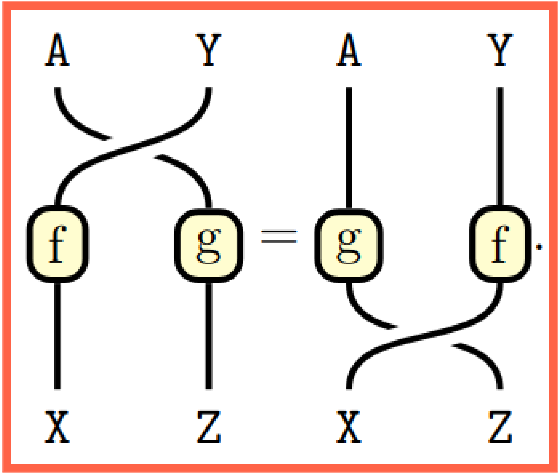
Braids first



A regular braid on five strands. Each arrow composes two further elements of B_5.

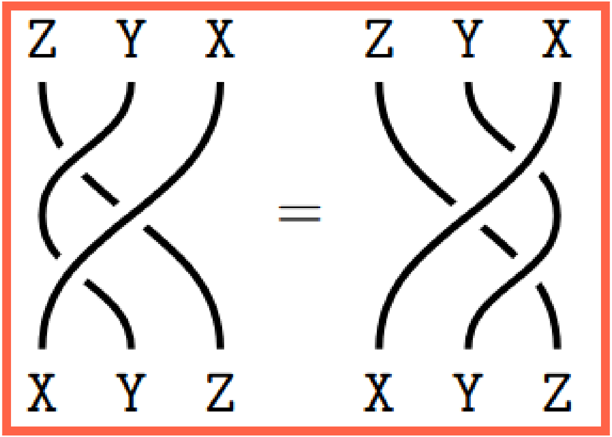
- Braids are generated by elementary crossings σ_i
- Knots and links are obtained by closing braids, so braids control knotting
- A braiding makes these crossings into morphisms in a category

The definition in one line



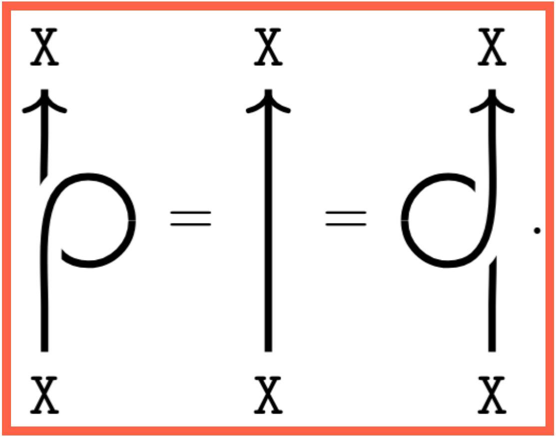
- A braiding is a natural isomorphism $\beta_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X$
- Naturality means swapping respects maps $f : X \to Y$ and $g : Z \to A$
- Coherence means swapping inside longer tensor products is unambiguous

The axiom: Reidemeister III / Yang–Baxter



- The fundamental braid move is Reidemeister III move, i.e. sliding one crossing past another
- Algebraically this is the braid relation $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$
- Categorically this is encoded by the hexagon identities for c

What braiding enables (next steps)



- A braiding gives braid group actions on tensor powers $X^{\otimes n}$ by local swaps
- If $\beta_{Y,X} \circ \beta_{X,Y} = id$, then the braiding is symmetric and crossings cancel
- With duals and a twist (later), braiding leads to link invariants from categorical data

Thank you for your attention!

I hope that was of some help.