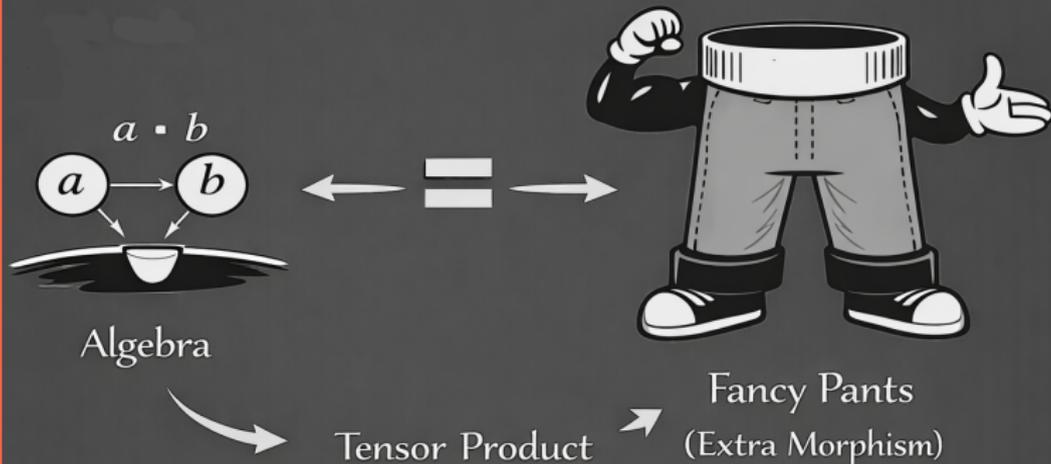


What is...quantum topology - part 23?

Or: Braided categories 2 from Chapter 5

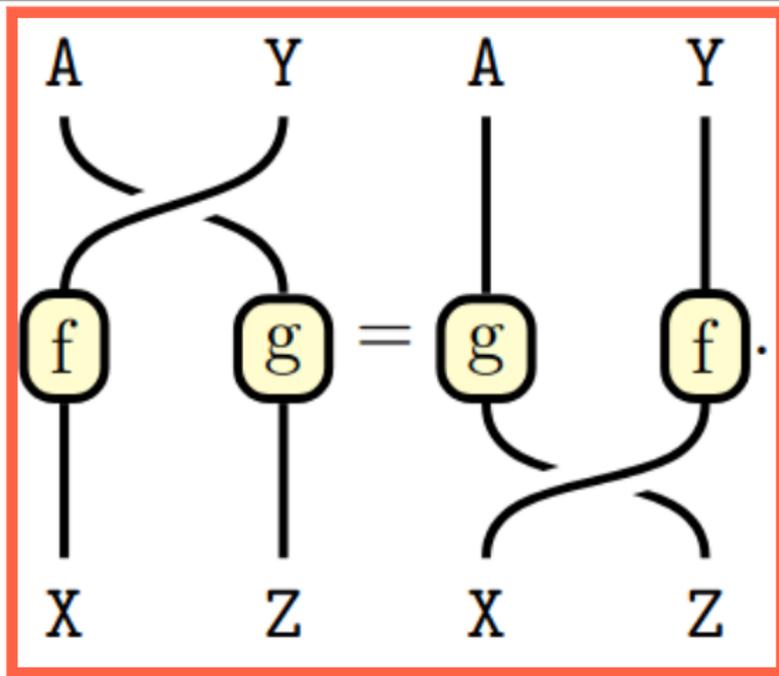
Braiding as commutativity

A Monoidal Category =
An Algebra Wearing Fancy Pants!



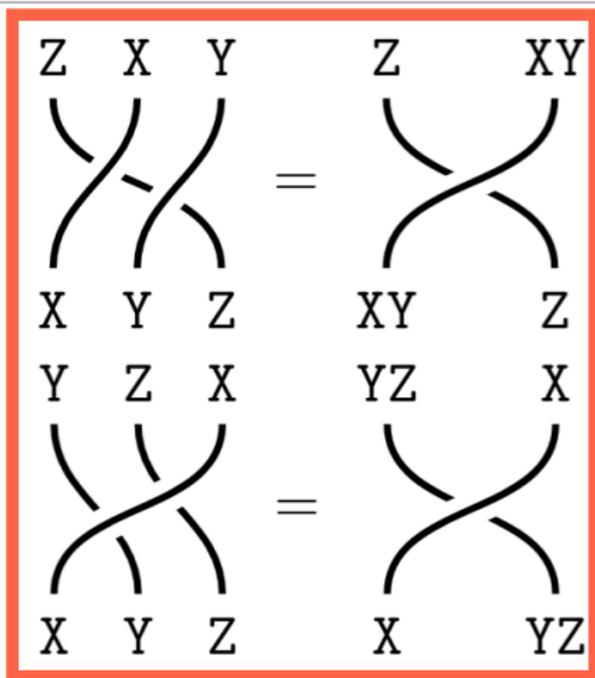
- ▶ A monoidal category is an algebra wearing fancy pants (later: addition)
- ▶ The monoidal product \otimes plays the role of multiplication, with unit object $\mathbb{1}$
- ▶ So the next question is how to express commutativity for \otimes

Commutativity needs a swap map



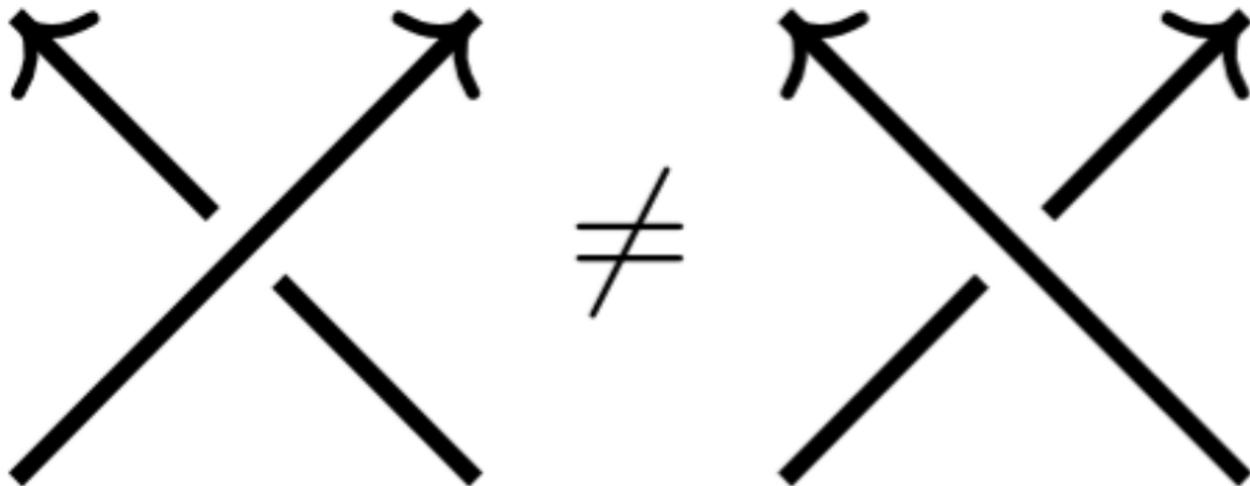
- ▶ In an algebra, commutativity means $xy = yx$ for all elements x, y
- ▶ In a monoidal category this becomes an iso. $\beta_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X$
- ▶ We require $\beta_{X,Y}$ to be natural in both X and Y

Why we need axioms (coherence)



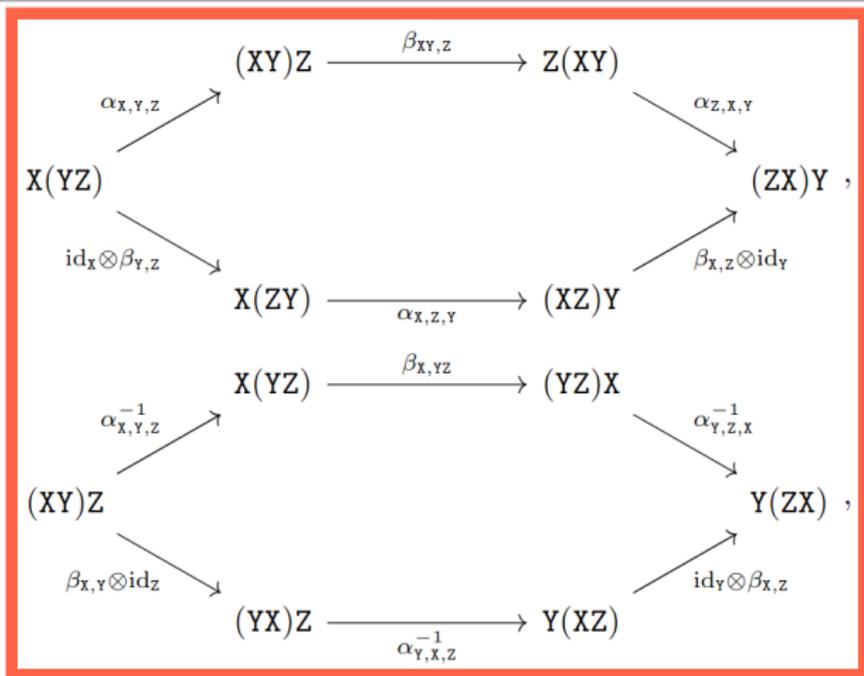
- ▶ There are **two** ways to move X past $Y \otimes Z$ using c (and associators)
- ▶ Commutativity should **not** depend on parenthesising choices
- ▶ The coherence requirement is exactly the **hexagon** condition

Symmetric vs braided commutativity



- ▶ If $\beta_{Y,X} \circ \beta_{X,Y} = id_{X \otimes Y}$, then swapping twice does nothing
- ▶ This is the **symmetric** case \iff usual commutative algebras
- ▶ If not, then this is the **“quantum”** setting

Definition: braided monoidal category



- ▶ A **braiding** is a natural family of isomorphisms $\beta_{X,Y} : X \otimes Y \xrightarrow{\sim} Y \otimes X$
- ▶ We have **Hexagon equation one** : top commutes
- ▶ We have **Hexagon equation two** : bottom commutes

Thank you for your attention!