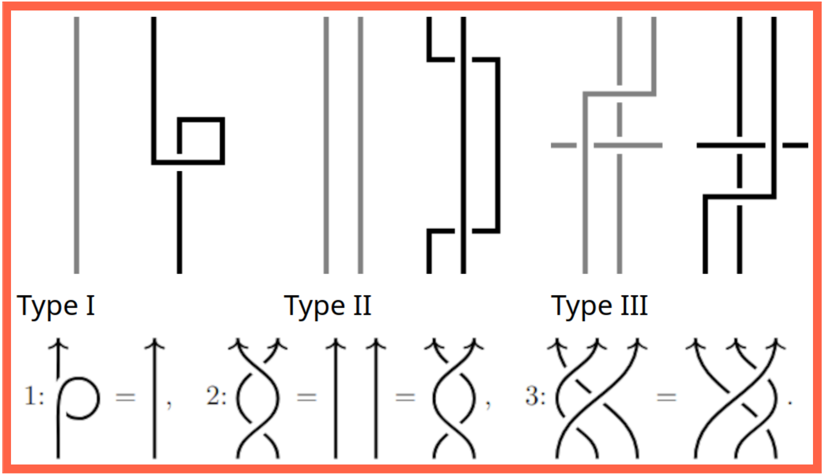
# What is...quantum topology - part 25?

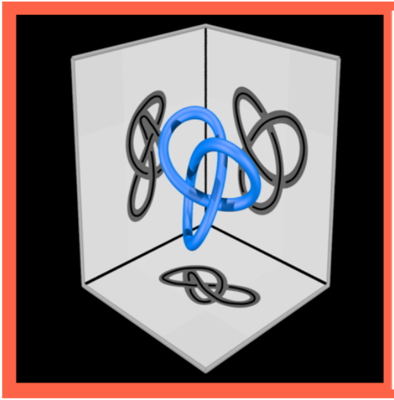
Or: Braided categories 4 from Chapter 5

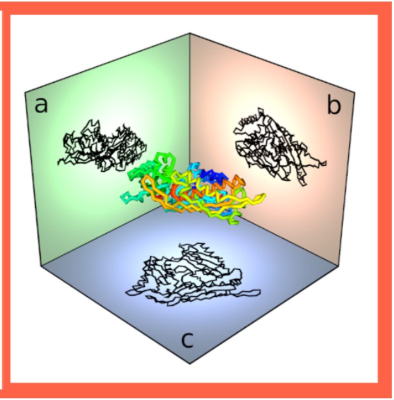
## Why Reidemeister moves matter




- A knot or tangle lives in three dimensions, but we usually draw it in the plane
- So we need local diagrammatic moves that do not change the topology
- Reidemeister's theorem: these are exactly planar isotopies plus moves 1–3

## The topological Reidemeister theorem



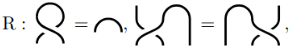



- Two tangles in three-space are isotopic iff their projections are related by allowed diagram moves
- These moves are the three Reidemeister moves together with ordinary planar deformations
- So topology can be tested by a finite local calculus on pictures

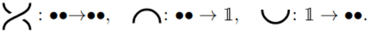
# Algebraic models: qBr and oqBr

**Example 5F.1.** The ***(generic) quantum Brauer category* qBr** is the braided pivotal category generated by one self-dual object • with relations

(5F-2)



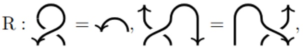
including mirrors, with structure maps



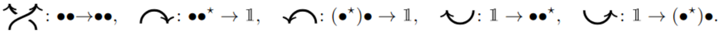
(Note that several relations come for free from "the braided pivotal category generated by".) ◇

**Example 5F.3.** The ***(generic) oriented quantum Brauer category* oqBr** is the braided pivotal category generated by one object • with relations

(5F-4)



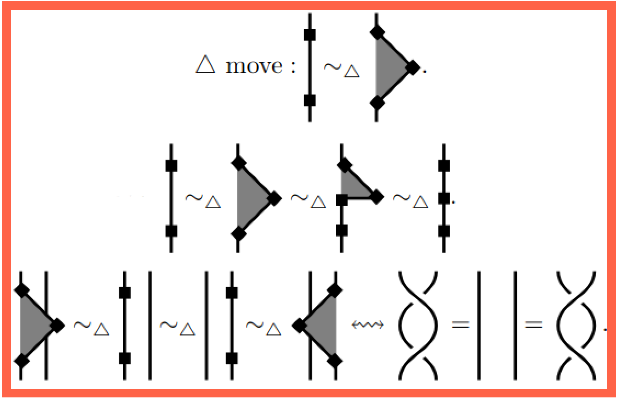
including mirrors, with structure maps



(As in Example 5F.1, several relations are hidden in the definition.) ◇

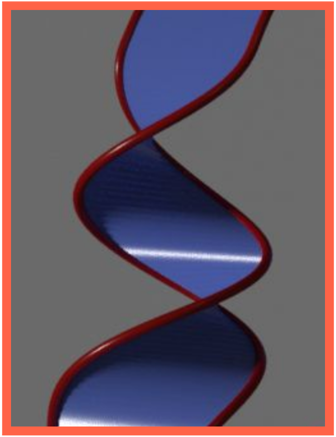
- qBr and oqBr are generated by strands, crossings, cups and caps
- Their relations encode the Reidemeister-type rules diagrammatically
- Slogan: replace tangles by an algebraic presentation of the same calculus

## The categorical Reidemeister theorem




- There are braided rigid functors $qR : qBr \to 1Tan$ and $oqR : oqBr \to 1State$
- They are dense, fully faithful, hence these algebraic categories recover tangles and states
- So the topological world and the categorical world are the same calculus

## Why this theorem is a big deal



- Once the diagrammatic calculus is understood, many topological questions become algebraic manipulations
- Later we can impose extras, like twists and ribbons, to build invariants
- This is the point where quantum topology without topology really starts

**Thank you for your attention!**