

What is...quantum topology - part 26?

Or: Braided categories 5 from Chapter 5

From braidings to twists

The diagram shows two equations for twists in a braided pivotal category. The first equation, $t_X =$, is followed by a diagram of a right twist: a vertical line starts at an object X at the bottom, goes up, forms a loop to the right, and then continues up to another X at the top. The second equation, $t^X =$, is followed by a diagram of a left twist: a vertical line starts at an object X at the bottom, goes up, forms a loop to the left, and then continues up to another X at the top. Both diagrams are enclosed in a red rectangular border.

- ▶ In a braided pivotal category, every object has right and left twists
- ▶ Diagrammatically, a twist is a curl built from braiding, cups and caps
- ▶ So braiding + duality naturally produces the Reidemeister 1 type data

Three versions of Reidemeister 1

First, the *(classical) Reidemeister 1 moves* are

(5H-2)

Second, the *ribbon equation* is

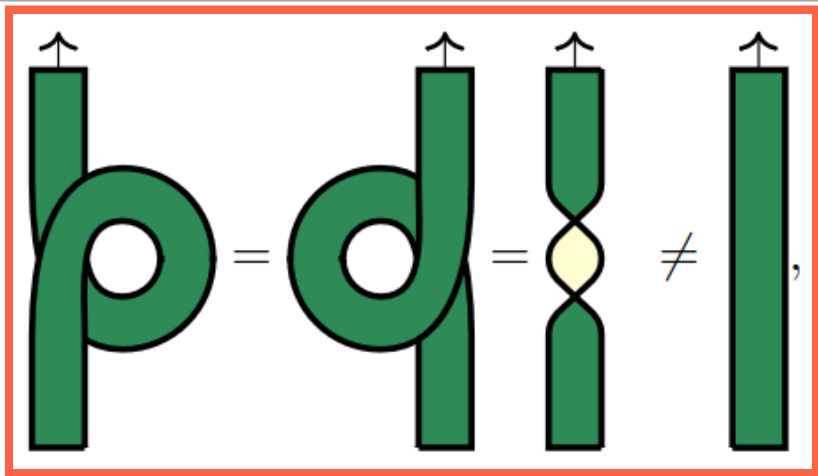
(5H-3)

Finally, the *framed Reidemeister 1 moves* are

(5H-4)

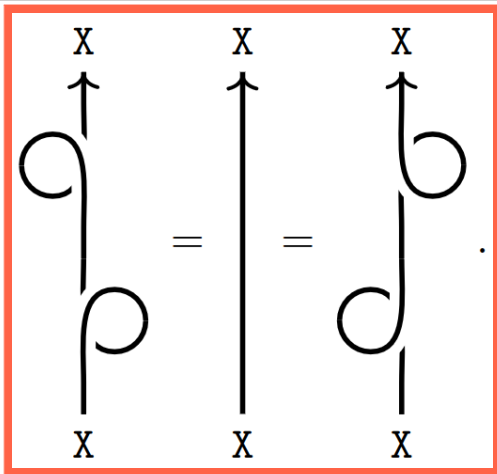
- ▶ The **classical Reidemeister 1** move says a curl simply disappears
- ▶ The **ribbon equation** identifies the left and right twists
- ▶ The **framed Reidemeister 1** move replaces a curl by the corresponding twist

Why these notions differ



- ▶ For vector spaces the classical Reidemeister 1 holds
- ▶ But sometimes e.g. ribbon holds but classical Reidemeister 1 fails
- ▶ So the distinction between braided, framed and ribbon is genuinely real

What always holds



- ▶ In any braided pivotal category, the framed Reidemeister 1 relation holds
- ▶ This comes from the naturality and sliding properties of the twists
- ▶ Hence twists are not a gimmick but built into the braided pivotal calculus

Ribbon categories



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- ▶ A ribbon category is a braided pivotal category satisfying the ribbon equation
 - ▶ This is the right algebraic setting for knot invariants
 - ▶ So: ribbon structure is where the topology comes back in

Thank you for your attention!