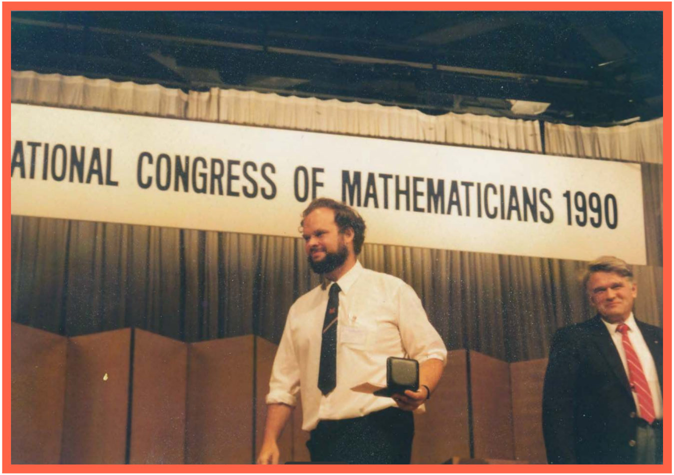
What is...quantum topology - part 27?

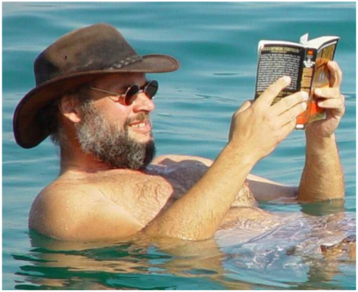
Or: Braided categories 6 from Chapter 5

A very short history I



- Knot theorists first used invariants from algebraic topology
- The Jones polynomial appeared ≈1985 and changed the subject overnight
- It was strong, computable and surprisingly mysterious

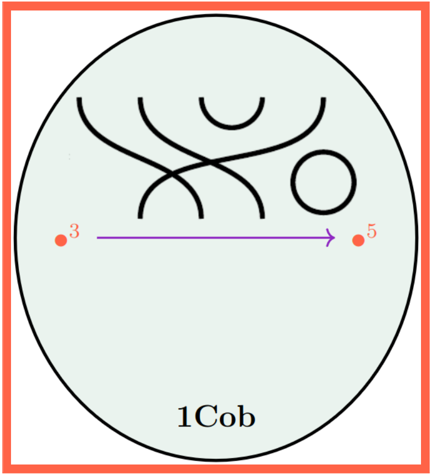
A very short history II



V. Jones

- Very soon people realised that these invariants come from algebraic and categorical structures
- Quantum groups, braided categories and topological quantum field theory all entered the story in a rather dramatic way
- This is the point: categories become a machine for invariants

Why ribbon categories matter



- A ribbon category the structure needed to read closed diagrams consistently
- So braidings, duals and twists were not decoration, but the input for topology
- The payoff is that knots and links now produce honest numerical invariants

Brauer and friends

5L. **Summary of the interplay between topology and categorical algebra.** Some (the most important) Brauer categories we have seen are summarized in the following table.

	monoidal	braided	pivotal	symmetric	self-dual •	Reidemeister 1	topology
Br	**Y**	**Y**	**Y**	Y	Y	Y	**1Cob**
qBr	**Y**	**Y**	**Y**	N	Y	Y	**1Tan**
oqBr	**Y**	**Y**	**Y**	N	N	Y	**1State**
orqBr	**Y**	**Y**	**Y**	N	N	N	**1Ribbon**

We leave it to the reader to fill in all the various versions using the adjectives "oriented", "quantum" and "ribbon". Let us use the placeholder _, which can be filled in with these adjective.

The point is that they are all equivalent to their topological incarnations while "free XYZ with properties ABC". Thus, we define:

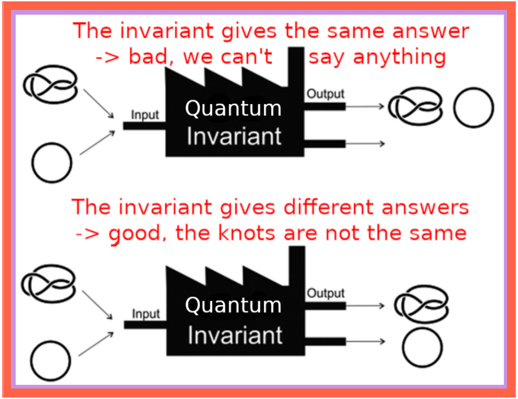
A ***quantum invariant*** Q is a structure preserving functor

$$Q\colon \_\mathbf{Br} \to \mathbf{C},$$

where **C** is "a linear algebra like category".

- Brauer-type categories model different flavors of topology
- The extra structure (braiding etc.) tells us which topology we see
- Categorical algebra and topology are two sides of the same graphical story

A definition (my take)



- A quantum invariant should be a structure-preserving functor

$$Q\colon \text{“some Brauer-thing”} \to \mathbf{C}$$

- Here $\mathbf{C}$ is some linear algebra-like category where we can actually compute
- The next lectures are about making this slogan precise and useful

Thank you for your attention!