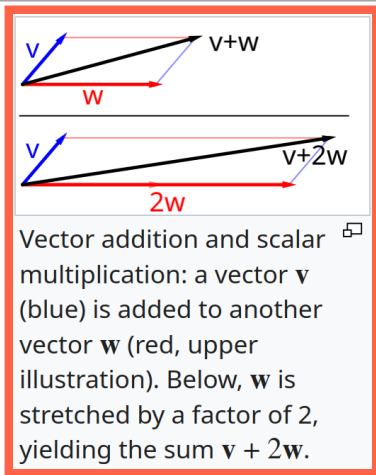


What is...quantum topology - part 29?

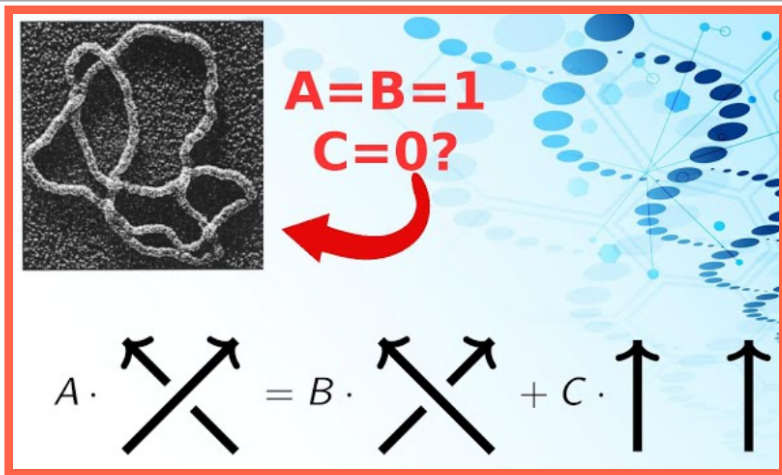
Or: Additive categories 2 from Chapter 6

Arrows become linear algebra



- ▶ In a linear category, each hom set is not just a set, but a **module**
- ▶ So parallel arrows can be added, subtracted and **rescaled**
- ▶ Composition is required to be bilinear: algebra now respects **gluing**

Why coefficients enter diagrams



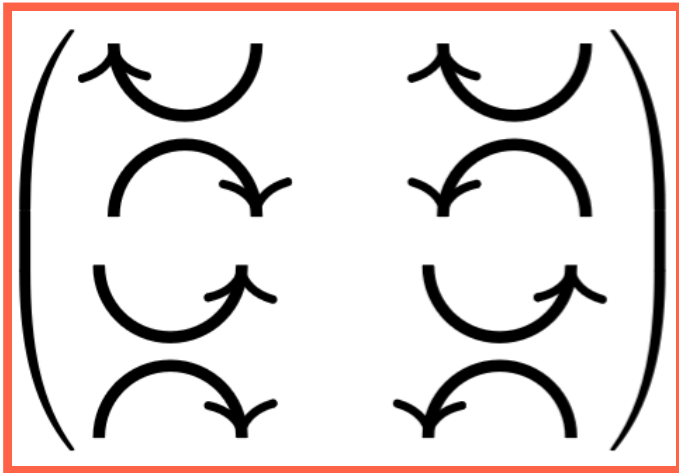
- ▶ Before, a diagram was one arrow; now we may take formal sums
- ▶ Example A skein relation says one picture = a linear combination of pictures
- ▶ This is the first moment where topology starts to look like calculation

Additive categories

$$A \oplus B = \begin{bmatrix} a_{11} & \cdots & a_{1n} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & b_{p1} & \cdots & b_{pq} \end{bmatrix}$$

- ▶ Additive Arrows \mathbb{Z} -add, there is a zero object, and finite direct sums exist
- ▶ A direct sum is both a product and a coproduct: one object doing two jobs
- ▶ Inclusions and projections are the handles that let us move in and out

Matrices without choosing coordinates

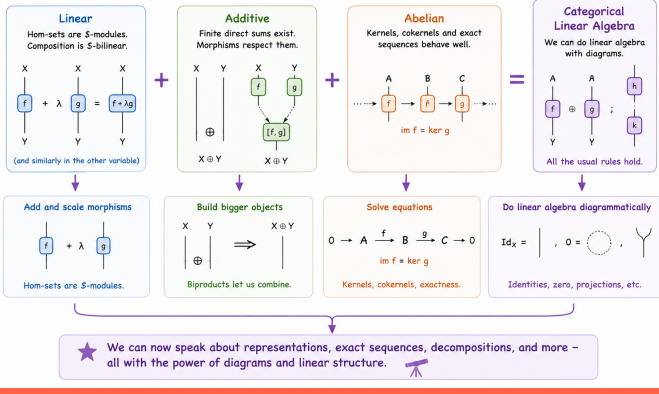


- ▶ A map out of a direct sum can be recorded by its components
- ▶ Thus maps between direct sums = matrices whose entries are morphisms
- ▶ Composition becomes matrix multiplication, but now inside a category

Why quantum topology wants this

Linear + Additive + Abelian = Categorical Linear Algebra

We now have a setting where linear algebra lives inside categories.



- ▶ The tangle world is geometric, but our target categories are usually **linear**
- ▶ Linear combinations let relations etc. become **honest algebra**
- ▶ Moral: additivity is the bridge from pictures to **representation theory**

Thank you for your attention!