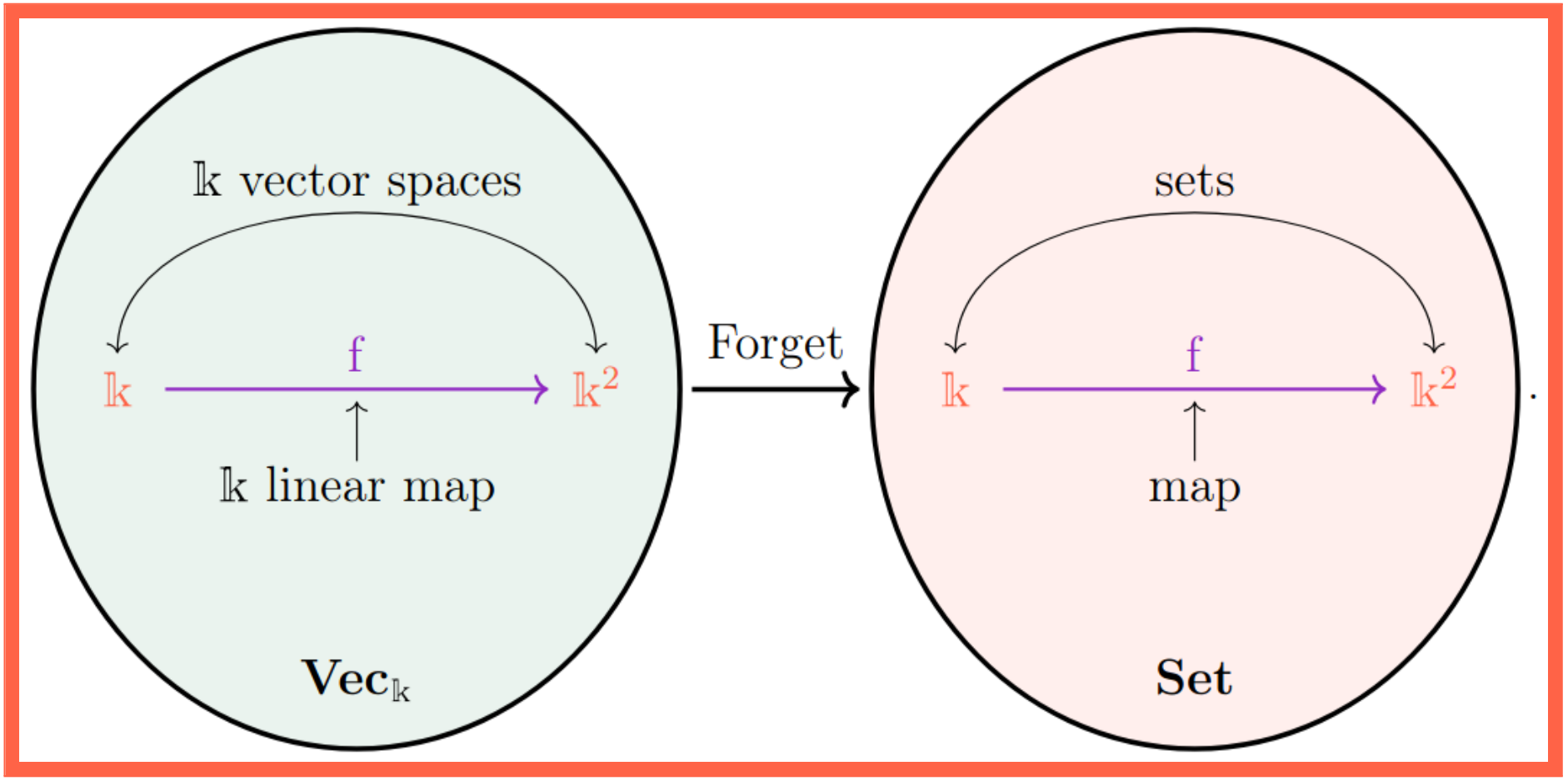
What is...quantum topology - part 5?

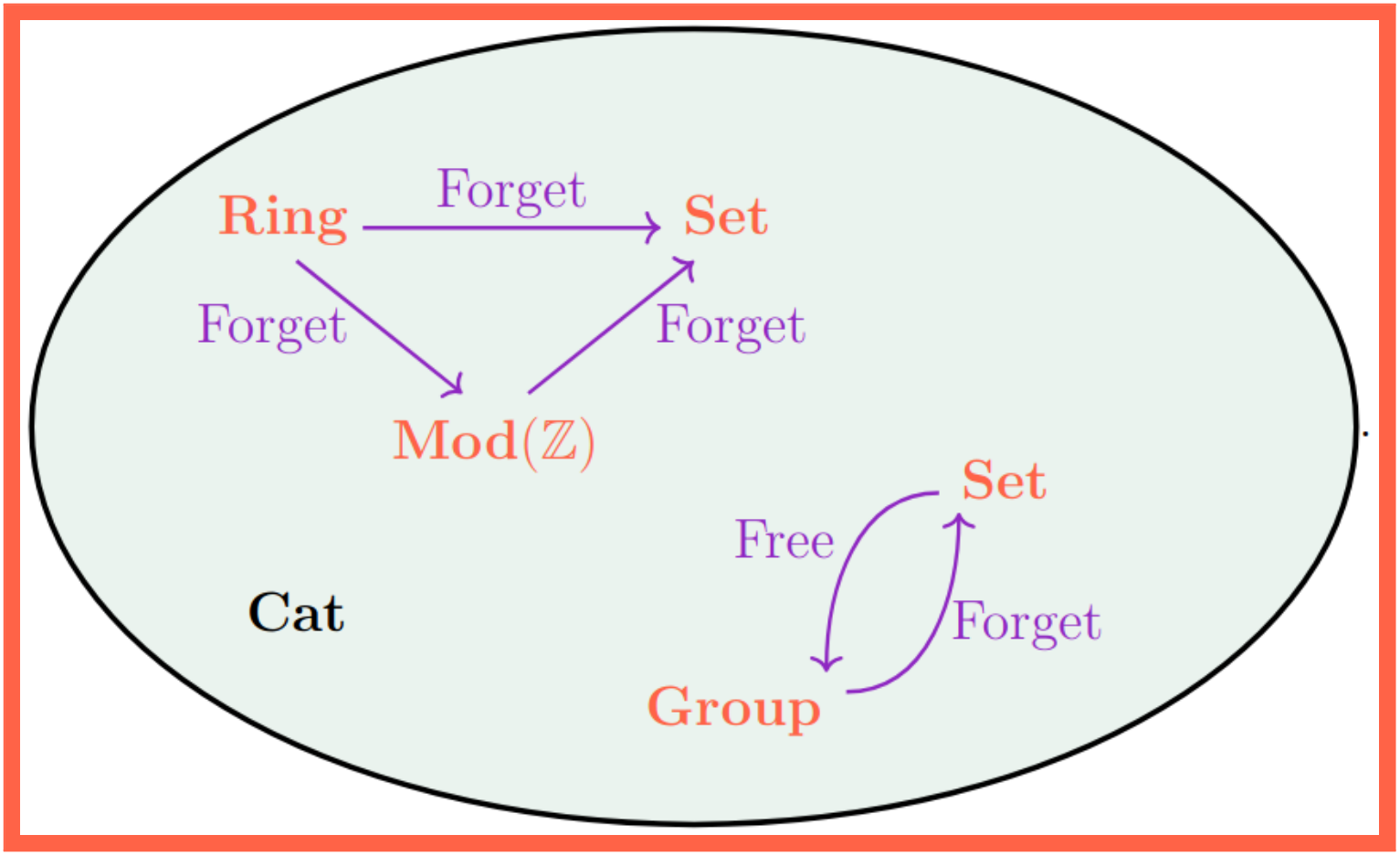
Or: Categories 3 from Chapter 1

The(!) examples of a category



- Above The category of sets (right) and the category of vector spaces (left)
- These are the same up to linearization
- In QT we like linear thingies so we prefer $\mathrm{Vec}_{\mathbb{k}}$

The(!) example of a category



- Above The category of categories
- In some sense category theory is self-referring
- Slogan Everything is a category, so focus on relations between objects

Another important example in QT

```
> G:=SymmetricGroup(3);
> X:=CharacterTable(G);
> X

   ----result----
> Character Table of Group G
>  -------------------------
>
>
> -----------------
> Class |   1  2  3
> Size  |   1  3  2
> Order |   1  2  3
> -----------------
> p  =  2   1  1  3
> p  =  3   1  2  1
> -----------------
> X.1   +   1  1  1
> X.2   +   1 -1  1
> X.3   +   2  0 -1
```

- Above The category of group representations
- Or rather its decategorification : the character ring
- Most of QT is some version of 1Tan or the character ring of a group

For completeness: A formal definition

A category C is a quadruple $C = (Ob(C), \hom_C, id, \circ)$ consisting of:

- A class $Ob(C)$ of object
- For $X, Y \in Ob(C)$ a set $\hom_C(X, Y)$ of arrows
- For $X \in Ob(C)$ and identity arrow id_X
- A composition for $f : X \to Y$ and $g : Y \to Z$ denoted $gf = g \circ f : X \to Z$

such that:

- $\circ$ is associative
- id_X are identities
- the sets $\hom_C(X, Y)$ are pairwise disjoint

"Like a set with arrows"
"Like a group with multiple start points"
"Like a universe where relations=arrows matter"

What categories does QT study?

Notion	Categorification
Set	Category
Monoid	Monoidal category
Involutive monoid	Pivotal category
Commutative monoid	Braided category
Abelian group	Additive/abelian category
Ring/algebra	Fiat/tensor category
Group ring	Fusion category

- Categories categorify sets
- 1Tan has more structure than a plain category
- QT therefore looks at categorifications with more structure, e.g. as above

Thank you for your attention!

I hope that was of some help.