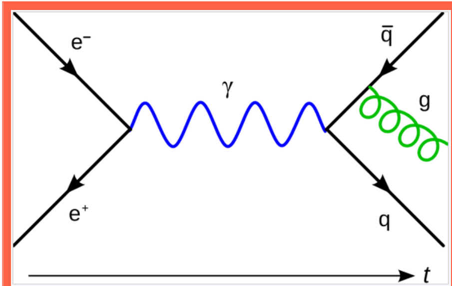
What is...quantum topology - part 7?

Or: Categories 5 from Chapter 1

Feynman diagrams

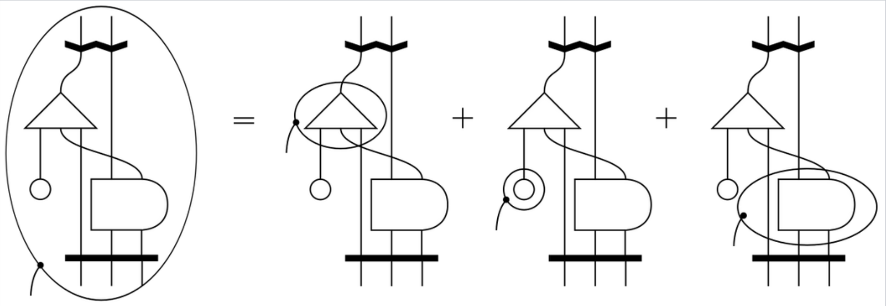


In this Feynman diagram, an electron (**e⁻**) and a positron (**e⁺**) annihilate, producing a photon (**γ**, represented by the blue sine wave) that becomes a quark–antiquark pair (quark **q**, antiquark **q̄**), after which the antiquark radiates a gluon (**g**, represented by the green helix).

I stole this picture from Wikipedi, page Feynman diagrams

- A diagrammatic calculus for computing amplitudes in QFT
- Lines represent particles; vertices represent interactions Typing
- Composition is by gluing: integrals over intermediate states Composition

Penrose graphical notation



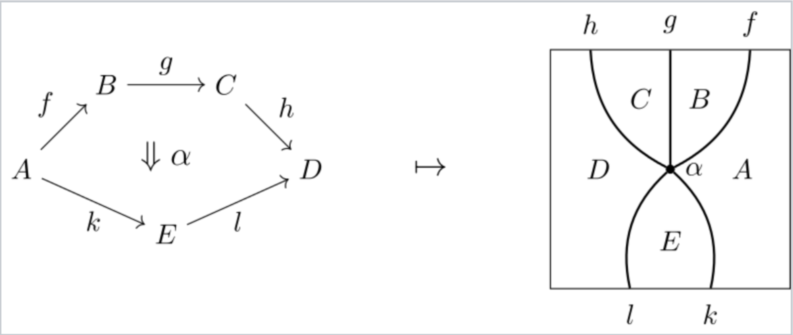
covariant derivative $12\nabla_a \left(\xi^f \, \lambda^{(d}_{fb[c} \, D^{e)b}_{gh]}\right)$

$$= 12 \left(\xi^f (\nabla_a \lambda^{(d}_{fb[c}) \, D^{e)b}_{gh]} + (\nabla_a \xi^f)\lambda^{(d}_{fb[c} \, D^{e)b}_{gh]} + \xi^f \lambda^{(d}_{fb[c} \, (\nabla_a D^{e)b}_{gh]})\right)$$

I stole this picture from Wikipedi, page Penrose notation

- A diagrammatic calculus for multilinear algebra
- Wires represent tensor indices; boxes represent tensors Typing
- Composition is tensor contraction via connected wires Composition

String diagrams



Duality between commutative diagrams (on the left hand side) and string diagrams (on the right hand side)

I stole this picture from Wikipedi, page String diagrams

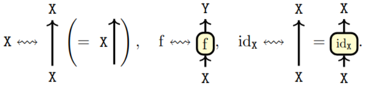
- A diagrammatic calculus for monoidal categories
- Wires are objects; nodes are morphisms Typing
- Composition is vertical (and later horizontal) stacking Composition

For completeness: A formal definition

1D. **Feynman diagrams.** We now discuss a convenient notation for categories, sometimes called ***Feynman*** (or ***Penrose*** or ***string*** or...) ***diagrams***, but we will also say *e.g.* ***diagrammatics***.

Given a category $\mathbf{C}$, we will denote objects $\mathtt{X} \in \mathbf{C}$ and morphisms $\mathrm{f} \in \mathbf{C}$ as

(1D-1)

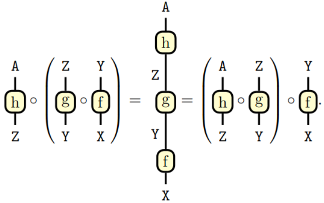


From now on, we use the convention from Convention 1A.3, meaning that we omit the orientations.

Remark 1D.2. This notation is “Poincaré dual” to the one $\mathrm{f}\colon \mathtt{X} \to \mathtt{Y}$ since, in diagrammatic notation, objects are strands and morphisms points, illustrated as coupons; see (1D-1). ◇

The composition is horizontal stacking, *i.e.*

(1D-3)



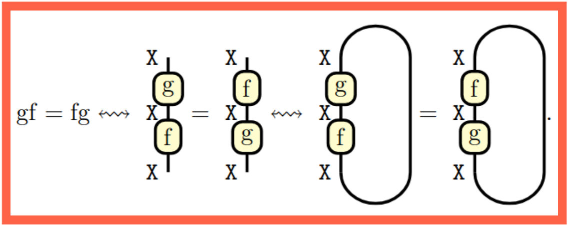
The formal rule of manipulation of these diagrams is:

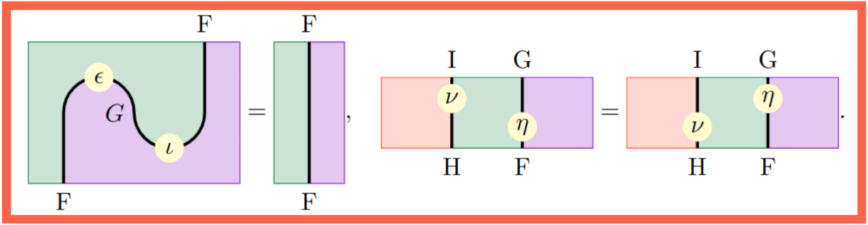
$$\text{“Two diagrams are equivalent if they are related by scaling.”} \tag{1D-4}$$

The following is (almost) immediate.

Theorem 1D.5. *The graphical calculus is consistent; i.e. two morphisms are equal if and only if their diagrams are related by* (1D-4).

String diagrams in QT





- Above With more structure at hand, string diagrams get very powerful
- At the moment they are rather one dimensional

Thank you for your attention!

I hope that was of some help.