What is...quantum topology - part 8?

Or: Categories 6 from Chapter 1

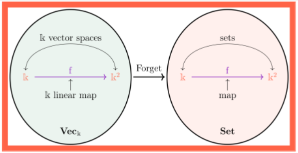
Only composition and identities

A category C is a quadruple $C = (Ob(C), \hom_C, id, \circ)$ consisting of:

- A class $Ob(C)$ of object
- For $X, Y \in Ob(C)$ a set $\hom_C(X, Y)$ of arrows
- For $X \in Ob(C)$ and identity arrow id_X
- A composition for $f: X \to Y$ and $g: Y \to Z$ denoted $gf = g \circ f: X \to Z$

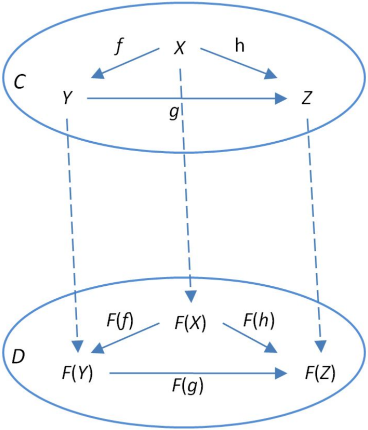
such that:

- $\circ$ is associative
- id_X are identities
- the sets $\hom_C(X, Y)$ are pairwise disjoint



- Recall In categories we have objects and morphisms
- Recall Morphisms can be composed and there are identities
- That's about it

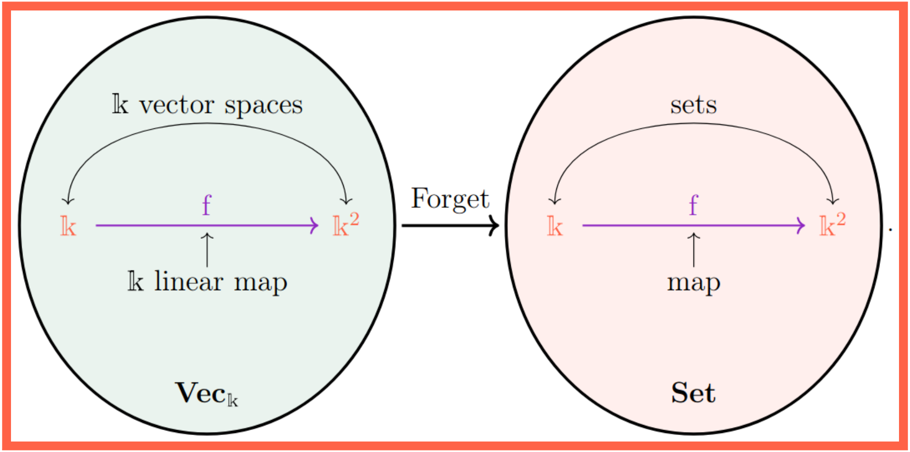
Preserving structure categorical



I stole this picture from somewhere, but I forgot from where...

- A functor F should be the correct map between categories
- F should associate objects to objects $X \mapsto F(X)$
- F should associate arrows to arrows $f \mapsto F(f)$ such that $F(gf) = F(g)F(f)$

Example: forgetting is easy



- Forgetful functor A functor from a rich category to a lean category
- Example Forget: $\mathbb{k}$Vec $\rightarrow$ Set
- This functor forgets that X is a $\mathbb{k}$-vector space and the f is $\mathbb{k}$-linear

For completeness: A formal definition

A functor F from C to D is a mapping that:

- associates each object X in C to an object $F(X)$ in D
- associates each arrow $f\colon X \to Y$ in C to an arrow $F(f)\colon F(X) \to F(Y)$ in D

such that:

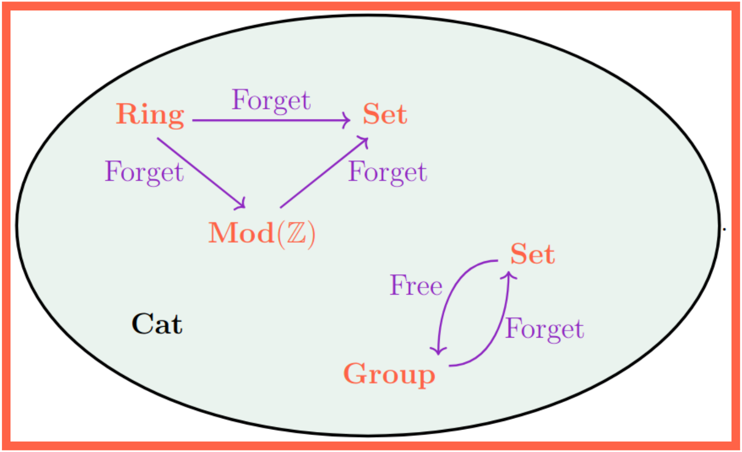
(a) $F(id_X) = id_{F(X)}$ "Unit goes to unit"

(b) $F(gf) = F(g)F(f)$ Composition is preserved

As usual:

- There is an identity functor $id_C\colon C \to C$
- Compositions of functors are functors
- Thus, $Fun(C, C)$ is a monoid
- Actually, $Fun(C, D)$ is a category but this will have to wait for a while

Cat of cats



- Categories themselves form a category Cat where arrows are functors
- Well, almost There are set-theoretical issues with Cat, but let us ignore that
- Slogan Everything forms a category, including categories

Thank you for your attention!

I hope that was of some help.