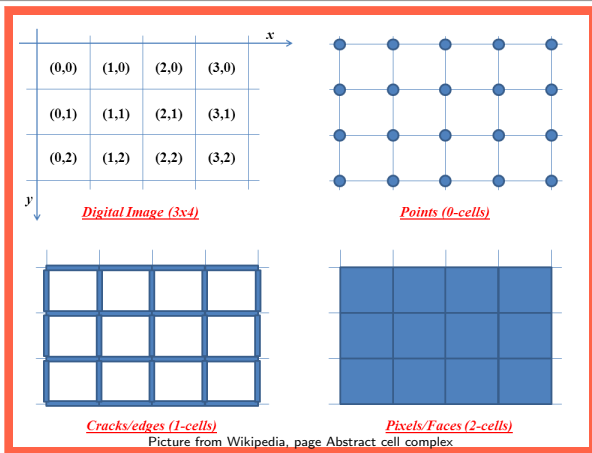


What is...quantum topology - part 9?

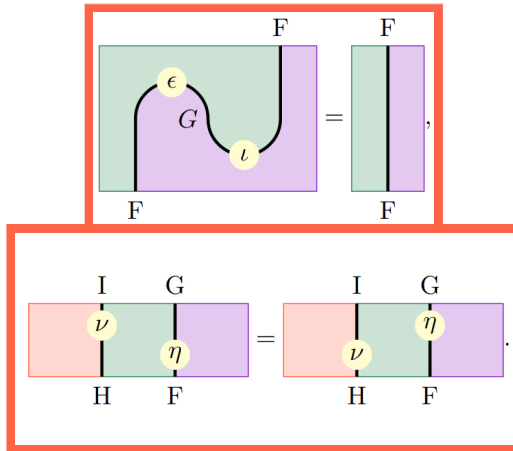
Or: Categories 7 from Chapter 1

Part 1 of 0-cells, 1-cells, 2-cells, ...



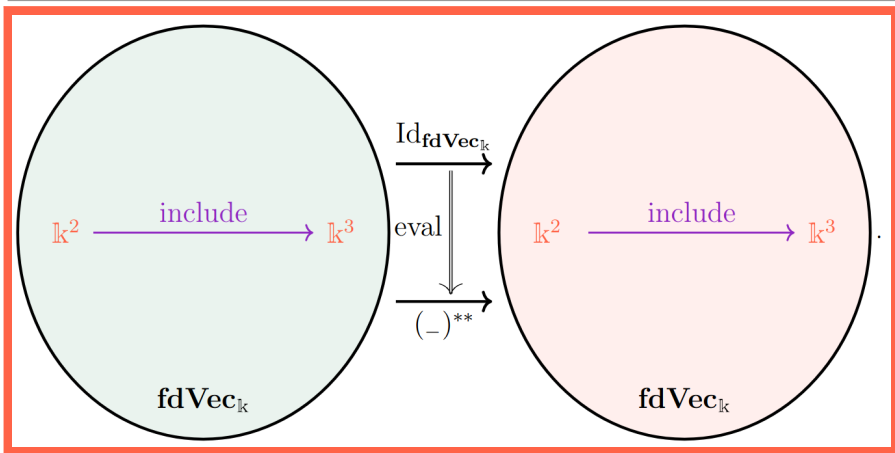
- Recall String diagrams so far give a one dimensional calculus
- Recall We want a two dimensional calculus
- So we need 0-cells, 1-cells, 2-cells

Part 2 of 0-cells, 1-cells, 2-cells, ...



- Let us color faces with categories
- Let us color strings with functors
- Missing: 0-cells Morphisms between functors

The double dual



- ▶ A vector space and its dual are isomorphic upon a choice of basis
- ▶ The double dual is isomorphic without choice of basis, by evaluation
- ▶ Evaluation is a morphism between the identity and the double dual

For completeness: A formal definition

A **nat trafo** $\eta: F \Rightarrow G$ is a mapping that:

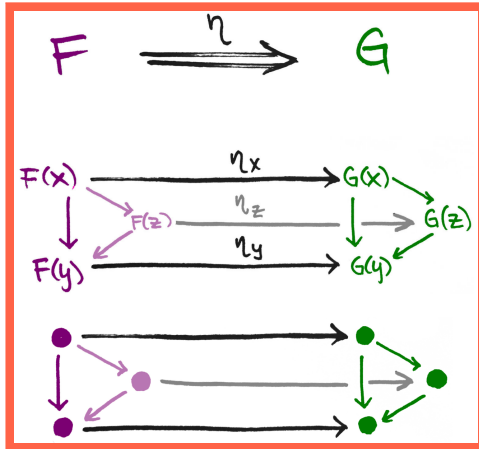
- associates each object X in C to an arrow $\eta_X: F(X) \rightarrow G(X)$ in D **Points \rightarrow Lines**
- such that $\eta_Y F(f) = G(f) \eta_X$ **Nat trafo square**

$$\begin{array}{ccccc} X & & F(X) & \xrightarrow{\eta_X} & G(X) \\ \downarrow f & & \downarrow F(f) & & \downarrow G(f) \\ Y & & F(Y) & \xrightarrow{\eta_Y} & G(Y) \end{array}$$

Here $F, G: C \rightarrow D$ are functors with same source and target categories

The tip of the iceberg: the arrow between nat trafos is called modification

Connecting fancier diagrams



I stole this picture from somewhere, but I forgot from where...

- **Functor** About commuting diagrams
- **Natural transformations (nat trafo)** About commuting polytopes

Thank you for your attention!

I hope that was of some help.