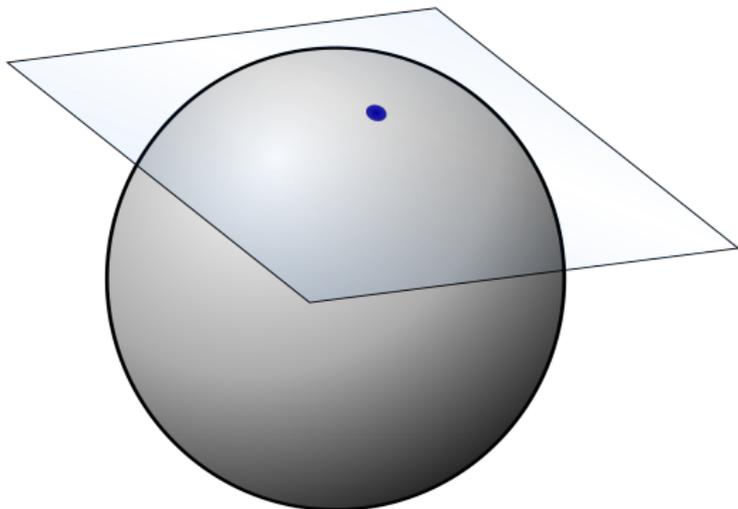


What is...representation theory?

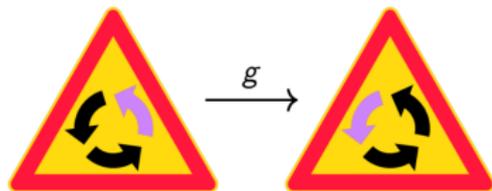
Or: Lets make life linear!

A linear perspective - part I

Tangent



Representation

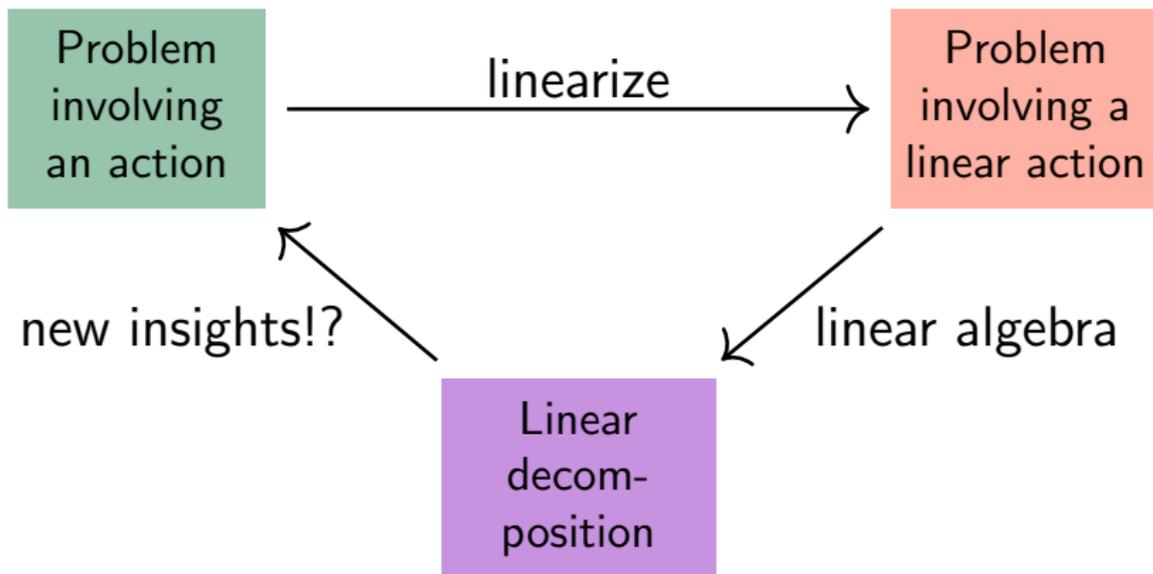


\leftrightarrow

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

- ▶ **Tangent** A linear approximation of a geometric object
- ▶ **Representation** A linear approximation of an algebraic object

A linear perspective - part II



-
- ▶ The study of actions (“symmetries”) is of fundamental importance
 - ▶ General actions are hard to study
 - ▶ The analog linear problem has often a satisfactory answer

The keywords – what representation theory (for example) studies

- ▶ Group representation basics
 - ▷ Schur's lemma
 - ▷ Maschke's Theorem
 - ▷ Character theory
 - ▷ ...
- ▶ Examples and applications
 - ▷ Fourier analysis on finite groups
 - ▷ Representations of symmetric groups
 - ▷ The representation ring
 - ▷ ...
- ▶ More representations
 - ▷ Representations over different fields
 - ▷ Representations of monoids
 - ▷ Categorical representations
 - ▷ ...

Direction one – applications in mathematics

From Burnside's book Theory of Groups of Finite Order

first
edition
~1897

It may then be asked why, in a book which professes to leave all applications on one side, a considerable space is devoted to substitution groups; while other particular modes of representation, such as groups of linear transformations, are not even referred to. My answer to this question is that while, in the present state of our knowledge, many results in the pure theory are arrived at most readily by dealing with properties of substitution groups, it would be difficult to find a result that could be most directly obtained by the consideration of groups of linear transformations.

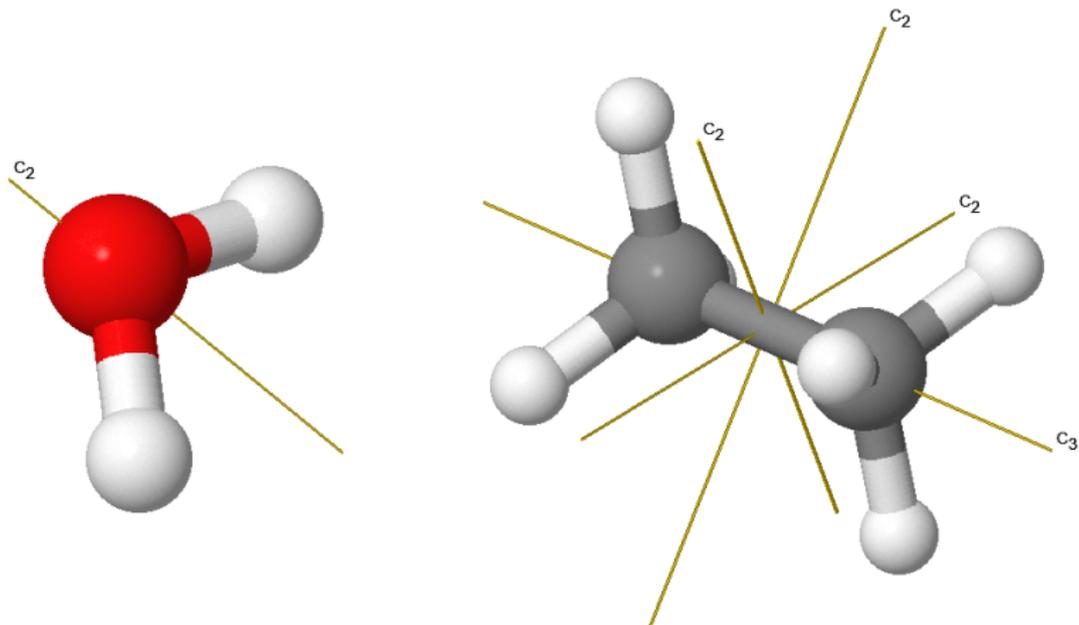
second
edition
~1911

VERY considerable advances in the theory of groups of finite order have been made since the appearance of the first edition of this book. In particular the theory of groups of linear substitutions has been the subject of numerous and important investigations by several writers; and the reason given in the original preface for omitting any account of it no longer holds good.

In fact it is now more true to say that for further advances in the abstract theory one must look largely to the representation of a group as a group of linear substitutions. There is

-
- ▶ Burnside's theorem A group of order $p^a q^b$ is solvable
 - ▶ Proof via representation theory ~1904
 - ▶ It took ~70 years to find a proof without the help of representation theory

Direction two – applications beyond mathematics



-
- ▶ **Chemistry** The symmetry of a molecule influence its chemical properties
 - ▶ How to study these? You guessed it: (also) via representation theory

Thank you for your attention!

I hope that was of some help.