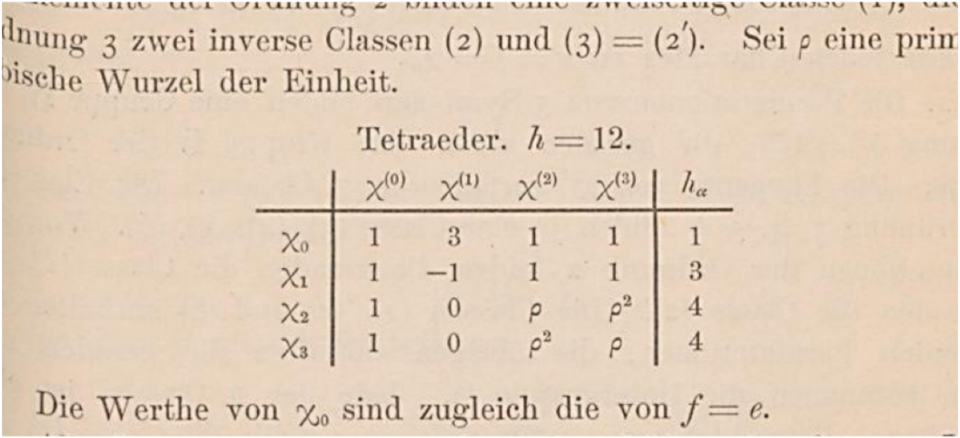
What is...a character table?

Or: The gist of the matter!?

The first ever published character table?



dnung 3 zwei inverse Classen (2) und (3) = (2′). Sei ρ eine prim
ische Wurzel der Einheit.

Tetraeder. $h = 12$.

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$	h_α
χ_0	1	3	1	1	1
χ_1	1	−1	1	1	3
χ_2	1	0	ρ	ρ^2	4
χ_3	1	0	ρ^2	ρ	4

Die Werthe von χ_0 sind zugleich die von $f = e$.

- Frobenius' character table of A_4 ~1896
- Character tables were around since the beginning of rep theory
- They contain basically all info about group reps in an efficient way

What a character table encodes – Part I

Alternating group A_4 of order 12

	χ_1	χ_2	χ_3	χ_4	#
C_1	1	3	1	1	1
C_2	1	-1	1	1	3
C_3	1	0	ρ	ρ^2	4
C_4	1	0	ρ^2	ρ	4

$\rho = \exp(2\pi i/3)$

- C_i = conjugacy classes; χ_i = simple characters over $\mathbb{C}$
- Square matrix in the middle = character values on the C_i
- Right column = size of the C_i
- Number of C_i = number of χ_i Char tables are squares
- Second row = dim of simple reps Char on id
- $\sum$ Squares second row = order of the group = sum of the right column

What a character table encodes – Part II

Alternating group A_4 of order 12

	χ_1	χ_2	χ_3	χ_4	#
C_1	1	3	1	1	1
C_2	1	-1	1	1	3
C_3	1	0	ρ	ρ^2	4
C_4	1	0	ρ^2	ρ	4

$\rho = \exp(2\pi i/3)$

- The rows are orthogonal, for example

$$(1,3,1,1) \perp (1,-1,1,1) \text{ since } 1\cdot 1 + 3\cdot(-1) + 1\cdot 1 + 1\cdot 1 = 0$$

- The columns are weighted orthogonal, for example

$$(1,1,1,1) \perp_{\#C_i} (1,1,\rho,\rho^2) \text{ since } 1\cdot 1\cdot 1 + 1\cdot 1\cdot 3 + 1\cdot\rho\cdot 4 + 1\cdot\rho^2\cdot 4 = 0$$

For completeness: A formal definition

Rows are labeled by simple characters, columns by conjugacy classes
The square matrix has the values of the characters on conjugacy classes

	(1)	(12)	(123)
χ_{triv}	1	1	1
χ_{sgn}	1	-1	1
χ_{stand}	2	0	-1

Careful: this is quite standard by now but transpose to Frobenius' notation

Properties of character tables over $\mathbb{C}$

- It is square meaning # simple chars = # conjugacy classes
- 1st column contains the simple dims; the sum of their squares is $|G|$
- The columns are orthogonal
- The rows are weighted orthogonal

Characters online

Character table of D_4

D_4: Dihedral group; $= He_2 = A\Sigma L_1(\mathbb{F}_4) = 2_+^{1+2}$ = square symmetries

class	1	2A	2B	2C	4	
size	1	1	2	2	2	
ρ_1	1	1	1	1	1	trivial
ρ_2	1	1	-1	1	-1	linear of order 2
ρ_3	1	1	1	-1	-1	linear of order 2
ρ_4	1	1	-1	-1	1	linear of order 2
ρ_5	2	-2	0	0	0	orthogonal faithful

```
G := Alt(4);
CT := CharacterTable(G);
CT;
```

```
Character Table of Group G
--------------------------


------------------------
Class |   1  2    3    4
Size  |   1  3    4    4
Order |   1  2    3    3
------------------------
p  =  2   1  1    4    3
p  =  3   1  2    1    1
------------------------
X.1   +   1  1    1    1
X.2   0   1  1    J -1-J
X.3   0   1  1 -1-J    J
X.4   +   3 -1    0    0
```

- It is nowadays very efficient to look up char tables online
- Conventions might vary, but its still fun A few links are in the description

Thank you for your attention!

I hope that was of some help.