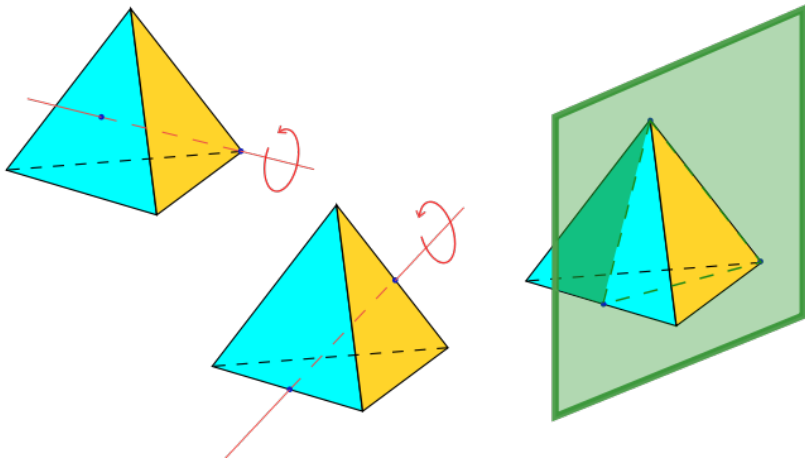


What are...Young diagrams?

Or: Representations of symmetric groups, part 1

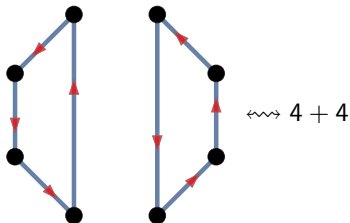
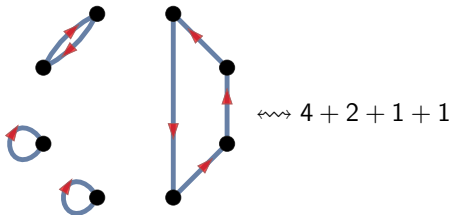
Reps of S_n

S_4 :



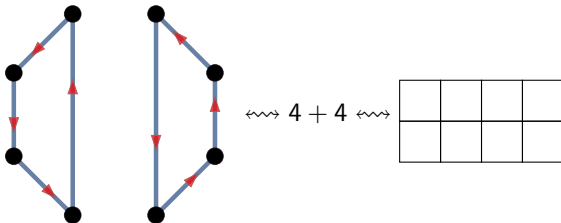
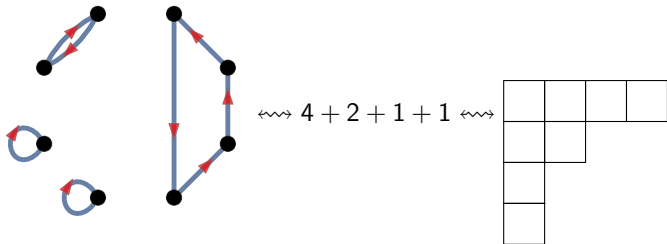
- ▶ Symmetric groups S_n are symmetry groups of $(n-1)$ simplices
- ▶ They and their reps appear everywhere in mathematics and beyond
- ▶ **Goal** Describe their representation theory by combinatorial means

Enter, partitions



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- ▶ Two permutations are conjugate if and only if they have the same cycle type
 - ▶ Cycle type = partitions and hence, partitions of $n \xleftrightarrow{1:1}$ simple S_n reps

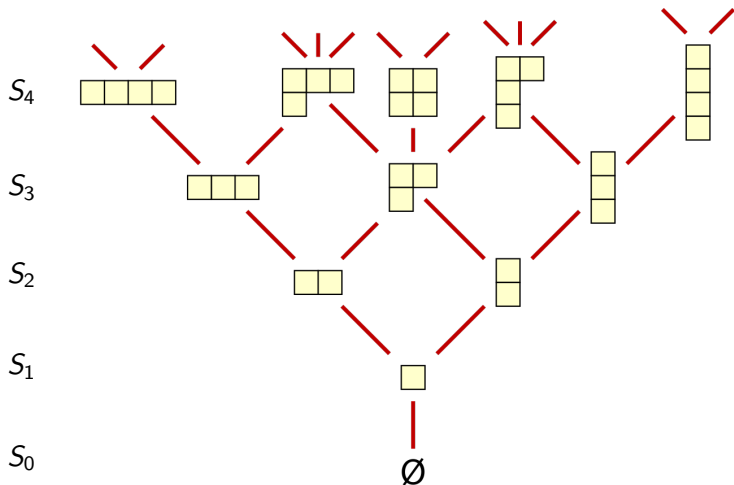
Young diagrams



- ▶ Young diagram = a finite collection of boxes arranged in left-justified nonincreasing rows
- ▶ Young diagrams are an efficient way to encode partitions

For completeness: A formal statement

The simple S_n reps/ \mathbb{C} are in 1:1 correspondence with Young diagrams with n boxes



Young tableaux

1	2	3
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$\leftrightarrow \dim = 1,$

1	2
3	

,

1	3
2	

$\leftrightarrow \dim = 2,$

1
2
3

$\leftrightarrow \dim = 1$

Class		1	2	3	
Size		1	3	2	
Order		1	2	3	

p	=	2	1	1	3
p	=	3	1	2	1

X.1	+	1	1	1	
X.2	+	1	-1	1	
X.3	+	2	0	-1	

- ▶ Young tableaux = a filling of a Young diagram with nonrepeating numbers $\{1, \dots, n\}$
- ▶ Standard Young tableaux = numbers increase along rows and columns
- ▶ **Theorem** The dims of the simple S_n reps/ \mathbb{C} are given by # std Young tableaux

Thank you for your attention!

I hope that was of some help.