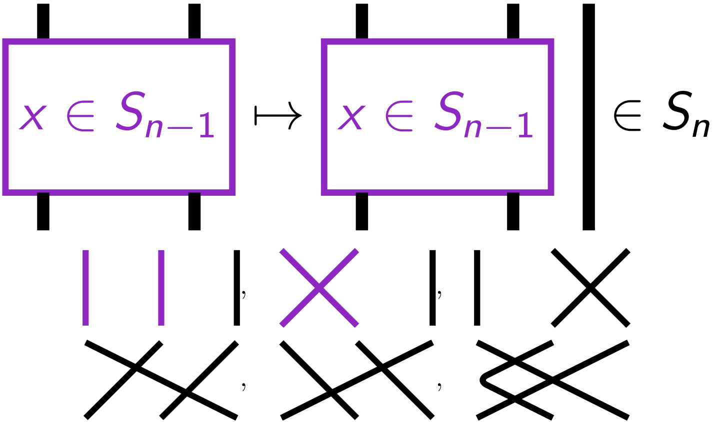
What is...the Young lattice?

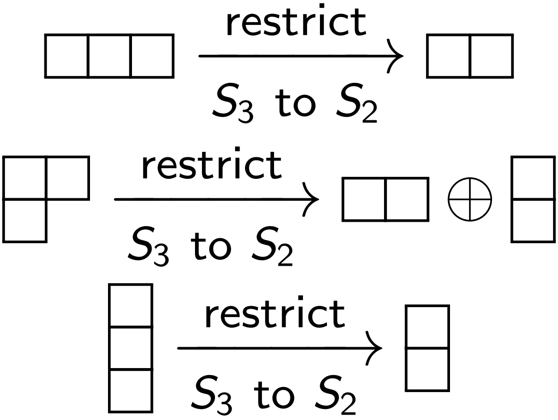
Or: Representations of symmetric groups, part 4

A tower of symmetric groups



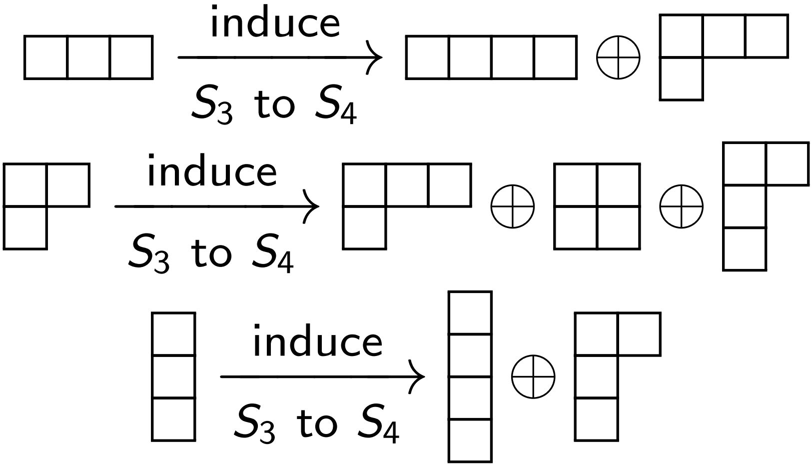
- S_{n-1} sits in S_n via adding strings
- We get a tower $\ldots \hookrightarrow S_{n-2} \hookrightarrow S_{n-1} \hookrightarrow S_n \hookrightarrow S_{n+1} \hookrightarrow \ldots$
- Use this sequence to say something about all S_n at once

Restriction



- Knowing *e.g.* the Specht modules one can find the restriction rule along the tower
- The restriction rule is: Sum of all ways to remove boxes

Induction

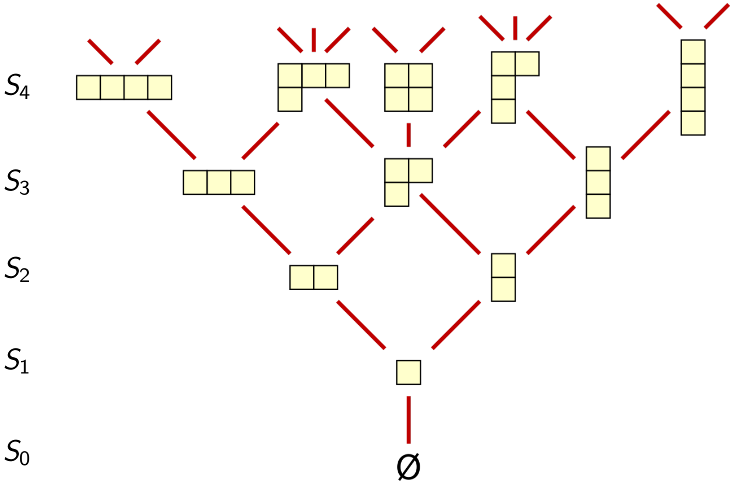


- Knowing *e.g.* Frobenius reciprocity one can find the induction rule along the tower
- The induction rule is: Sum of all ways to add boxes

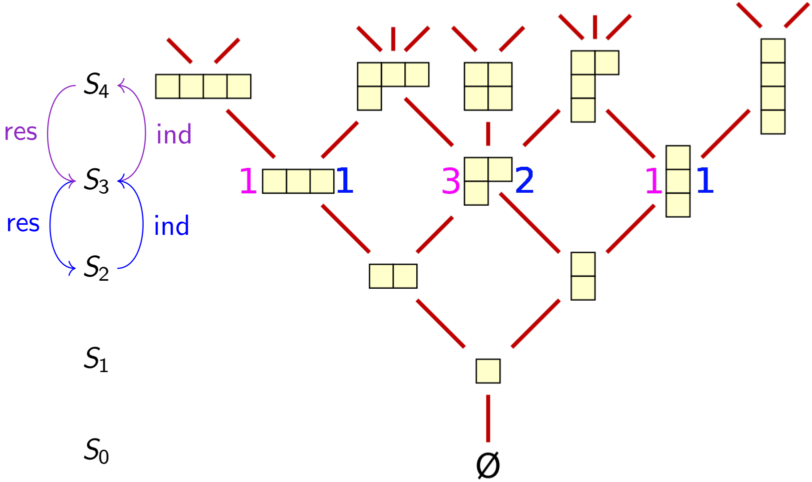
For completeness: A formal statement

The Young lattice describes induction and restriction for

$$\ldots \hookrightarrow S_{n-2} \hookrightarrow S_{n-1} \hookrightarrow S_n \hookrightarrow S_{n+1} \hookrightarrow \ldots$$



Symmetric groups know derivatives; well, kind of...



- The Leibniz rule $\partial_x x = x\partial_x + 1$
- The categorical Leibniz rule $\mathrm{Res} \circ \mathrm{Ind} = \mathrm{Ind} \circ \mathrm{Res} + \mathrm{id}$

Thank you for your attention!

I hope that was of some help.