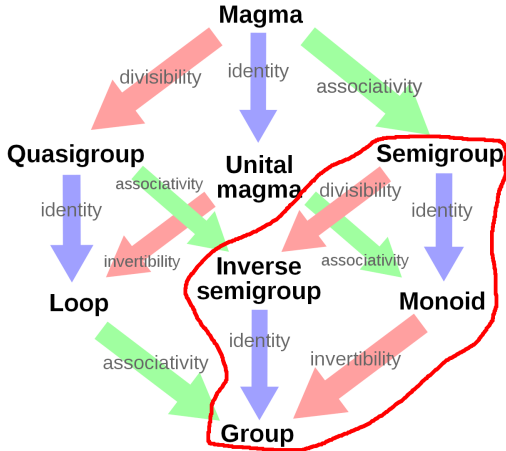


**What is...representation theory of monoids?**

---

Or: What to expect

# Matrix representations



- ▶ Adjoining identities is free For me “monoid=semigroups”
- ▶ Associativity  $\Rightarrow$  reasonable theory of matrix reps !?
- ▶ Southeast corner  $\Rightarrow$  reasonable theory of matrix reps !?

# Groups are kind of random...

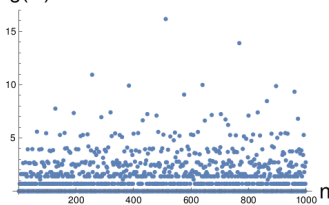
A000001 Number of groups of order  $n$ .  
(Formerly M0098 N0035)

0, 1, 1, 1, 2, 1, 2, 1, 5, 2, 2, 1, 5, 1, 2, 1, 14, 1, 5, 1, 5, 2, 2, 1, 15, 2, 2, 5, 4, 1, 4, 1, 51, 1, 2, 1, 14, 1, 2, 2, 14, 1, 6, 1, 4, 2, 2, 1, 52, 2, 5, 1, 5, 1, 15, 2, 13, 2, 2, 1, 13, 1, 2, 4, 267, 1, 4, 1, 5, 1, 4, 1, 50, 1, 2, 3, 4, 1, 6, 1, 52, 15, 2, 1, 15, 1, 2, 1, 12, 1, 10, 1,

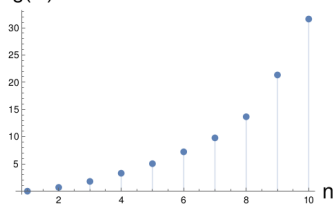
A058133 Number of monoids (semigroups with identity) of order  $n$ , considered to be equivalent when they are isomorphic or anti-isomorphic (by reversal of the operator).

0, 1, 2, 6, 27, 156, 1373, 17730, 858977, 1844075697, 52991253973742 ([list](#); [graph](#); [refs](#); [listen](#); [history](#);

Log( $\#$ )



Log( $\#$ )



- ▶ Groups are very special  $\Rightarrow$  nice rep theory
- ▶ Monoids have almost no structure, and there are zillions of them
- ▶ Unclear if monoid rep theory is “better” than general algebra rep theory Spoiler: It is!

## Groups vs. monoids – information loss



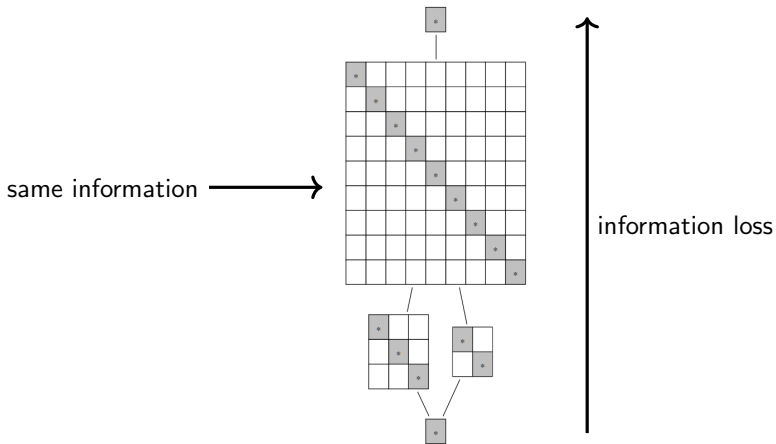
- ▶ Multiplication in a group preserves information
- ▶ Multiplication in a monoid destroys information in general
- ▶ MVP in (rep) theory of monoids Green cells measure information loss

# For completeness: **A** (primer of a) formal statement

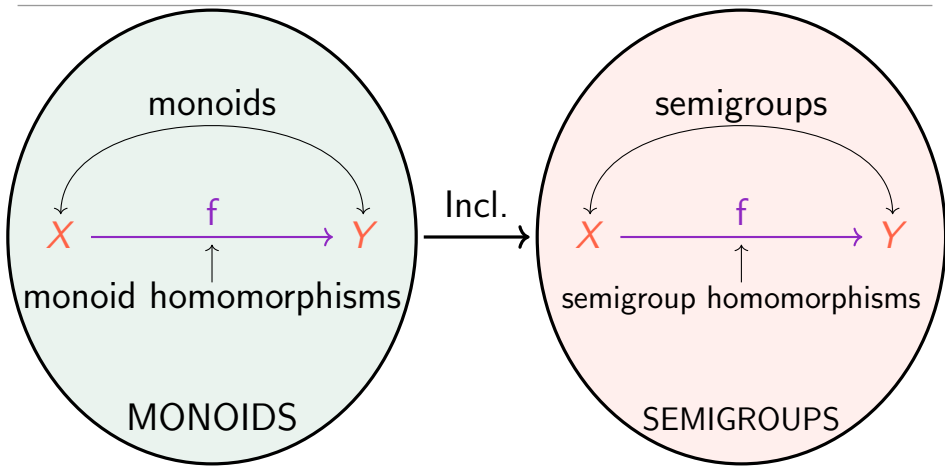
Clifford, Munn, Ponizovskii ~1940++:

The rep theory of a monoid is controlled by its maximal subgroups

Cells Pictures we will see all the time:



# "Monoids = semigroups"



- ▶ Assume we would not know that adjoining identities works
- ▶ The adjoint functor theorem shows that Inclusion has a left adjoint  $F$
- ▶ Define  $F(X)$  as "universal" way to adjoin an identity

**Thank you for your attention!**

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I hope that was of some help.