What is...the CMP theorem?

Or: Clifford–Munn–Ponizovskiĭ theorem a.k.a. H-reduction

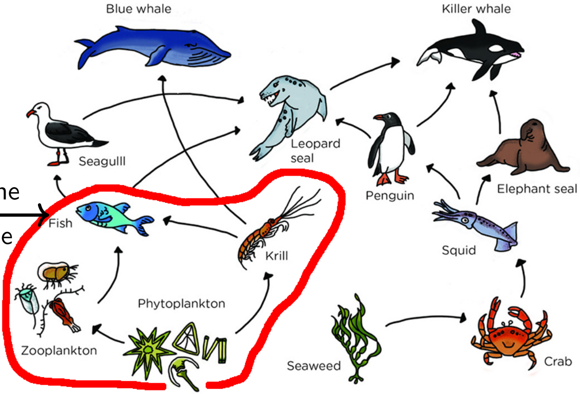
Cells in the theory of monoids

$\mathcal{J}_t$	a^3, a^4	$\mathcal{H}(e) \cong \mathbb{Z}/2\mathbb{Z}$
$\mathcal{J}_{a^2}$	a^2	
		$C_{3,2} = \langle a \vert a^{3+2} = a^3 \rangle$
$\mathcal{J}_a$	a	
$\mathcal{J}_b$	1	$\mathcal{H}(e) \cong 1$

- Cells order the monoid into equivalence classes of equal information
- Question Are cells useful to study monoid reps?
- Spoiler Simples $\leftrightsquigarrow$ "conjugacy classes (of $\mathcal{H}(e)$) + J-cells (apexes)"

Apex (predators)

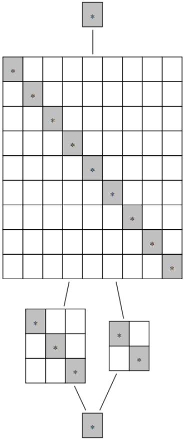
Apex of the red bubble



The simples for fish are not annihilated by fish, krill, zooplankton, phytoplankton

- J-cells are nicely ordered
- An apex is a maximal J-cell not annihilating a rep
- Theorem Simple monoid reps have a unique apex

Count per apex



- The J-cell apexes order simples
- Within one J-cell we have the H-cells
- **Theorem** The H-cells count simple of a fixed apex

For completeness: A formal statement

The Clifford–Munn–Ponizovskiĭ theorem a.k.a. H-reduction

- A J-cell is an apex $\Leftrightarrow$ it contains an idempotent
- Every idempotent J-cell contains a subgroup $\mathcal{H}(e)$
- There is a one-to-one correspondence

$$\left\{\begin{matrix}\text{simples with}\\ \text{apex } \mathcal{J}(e)\end{matrix}\right\} \xleftrightarrow{\text{one-to-one}} \left\{\begin{matrix}\text{simples of (any)}\\ \mathcal{H}(e) \subset \mathcal{J}(e)\end{matrix}\right\}$$

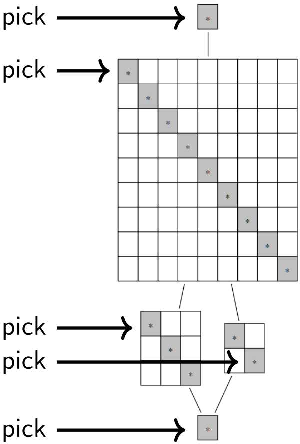
Reps of monoids are controlled by $\mathcal{H}(e)$-cells

$\mathcal{J}_t$	a^3, a^4	$\mathcal{H}(e) \cong \mathbb{Z}/2\mathbb{Z}$
$\mathcal{J}_{a^2}$	a^2	
$\mathcal{J}_a$	a	
$\mathcal{J}_b$	1	$\mathcal{H}(e) \cong 1$

$C_{3,2} = \langle a | a^{3+2} = a^3 \rangle$

We get $3 = 1 + 2$ simples/$\mathbb{C}$ for $C_{3,2}$

This is really powerful: reduce to H-cells



- Above: $\mathcal{H}(e)$ are shaded
- For simple reps we only ever need to consider one $\mathcal{H}(e)$ per apex
- Analogy For 1000 huge matrices, picking one element per matrix suffices

Thank you for your attention!

I hope that was of some help.