What is...a local-global principle?

Or: Small determines big

Hasse's local-global principle

Hasse's local-global principle roughly asks when the following are equivalent:

- A Diophantine equation is solvable in $\mathbb{Z}$ global
- A Diophantine equation is solvable modulo every p^k local $+$ in $\mathbb{R}$

When can local solutions be joined to a global solution?

A local-global principle in rep theory?

```
T := CharacterTable(AlternatingGroup(5));
Blocks(T,2);
DefectGroup(T[1],2);
```

```
[
    { 1, 2, 3, 5 },
    { 4 }
]
[ 2, 0 ]
Permutation group acting on a set of cardinality 5
Order = 4 = 2^2
    (1, 2)(3, 4)
    (1, 3)(2, 4)
```

Z/2Z x Z/2Z

e.g. $N_{A_4}(\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}) = \mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$ and only defect group is $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$

- Question What can subgroups tell us about group reps?
- Refined question What can p subgroups tell us about group reps?

Brauer's local-global principle I

```
T := CharacterTable(DihedralGroup(6));
Blocks(T,2);
DefectGroup(T[1],2);
DefectGroup(T[5],2);
```

```
[
    { 1, 2, 3, 4 },
    { 5, 6 }
]
[ 2, 1 ]
Permutation group acting on a set of cardinality 6
Order = 4 = 2^2
    (1, 5)(2, 4)
    (1, 4)(2, 5)(3, 6)
Permutation group acting on a set of cardinality 6
Order = 2
    (1, 4)(2, 5)(3, 6)
```

Z/2Z x Z/2Z

Z/2Z

e.g. $N_{D_6}(\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}) = \mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$ and only defect group is $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$

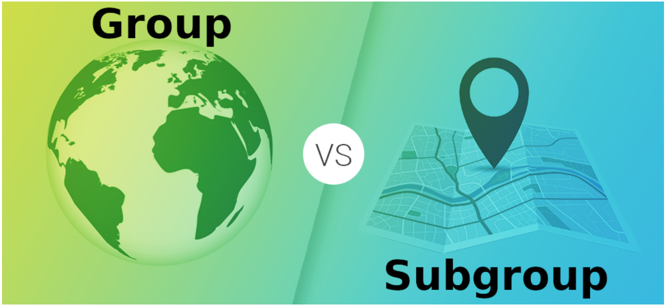
Brauer's local-global principle I says that there is a bijection between:

- Blocks of G with defect group D global
- Blocks of $N_G(D)$ with defect group D local

For completeness: A formal statement

We have Brauer's three main theorems:

- Brauer I The one from the previous slide
- Brauer II Roughly, an element t of prime order and a criterion for a block of $C_G(t)$ to correspond to a given block of G
- Brauer III A strengthening/version of I for the principal block



More local-global in rep theory

Brauer's main theorems are just the tip of the iceberg

There are many (often conjectural) local-global principles in rep theory, *e.g.*:

- Brauer's height-zero conjecture
- McKay conjecture
- Broué's abelian defect group conjecture
- More...

Thank you for your attention!

I hope that was of some help.