What is...tropical geometry - part 2?

Or: Tropical arithmetic

Tropical addition

Tropical Addition Table

$\oplus$	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	2	2	2	2	2	2
3	1	2	3	3	3	3	3
4	1	2	3	4	4	4	4
5	1	2	3	4	5	5	5
6	1	2	3	4	5	6	6
7	1	2	3	4	5	6	7

- Let us redefine $+ = \min$ (some people use max), e.g. $1 + 2 = 1$
- Observations $0 = \infty$; $+$ is associative and commutative
- Tropical I always write $\oplus$ instead of $+$

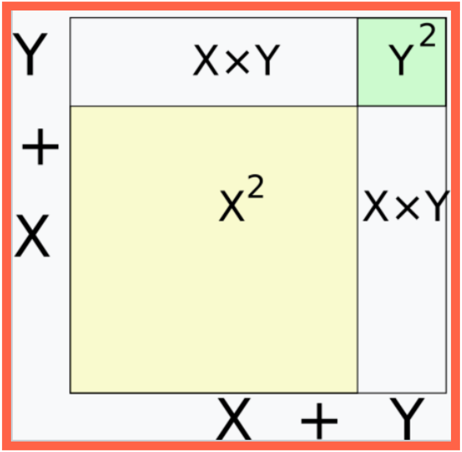
Tropical multiplication

Tropical Multiplication Table

$\otimes$	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10
4	5	6	7	8	9	10	11
5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13
7	8	9	10	11	12	13	14

- Let us redefine $\cdot = +$, e.g. $1 \cdot 2 = 3$
- Observations $1 = 0$; $\cdot$ is associative and commutative, and distributes over $+$
- Tropical I always write $\otimes$ instead of $\cdot +$

Freshperson's dream



- Standard arithmetic $(x+y)^2 = x^2 + 2xy + y^2 \neq x^2 + y^2$
- Tropical arithmetic $(x \oplus y)^2 = x^2 + y^2$ (fun exercise)
- Tropical I use standard abbreviations like $x^2 = x \otimes x$ etc.

For completeness: A formal statement

The tropical semiring (the tropicals) is

$$\mathbb{T} = (\mathbb{R} \cup \{\infty\}, \oplus, \otimes)$$

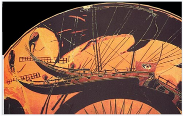
This is a (commutative) semiring, meaning:

(i) $\oplus, \otimes$ are unital, associative and commutative

(ii) $\infty \otimes x = x \otimes \infty = \infty$

(iii) $\oplus, \otimes$ distribute over one another

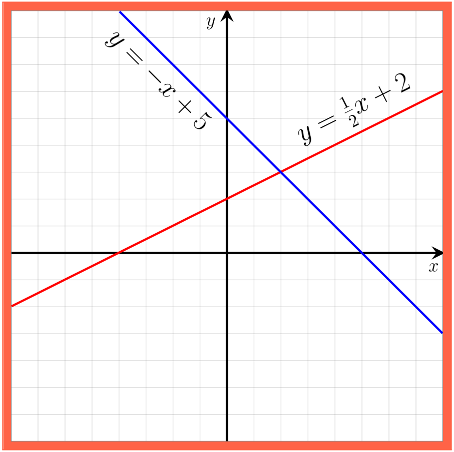
- There are other examples of semirings, but the tropicals will be the main player

Boolean semiring $\leftrightsquigarrow$ 

Aristotle: if a sea-battle will not be fought tomorrow, then it was also true yesterday that it will not be fought. But all past truths are necessary truths. Therefore, it is not possible that the battle will be fought.

- TG = geometry with ground "field" $\mathbb{T}$

Solving equations is difficult



- There is no direct minus
- Example $3 \oplus x = 12$ has no solution
- Catch Any form of linear algebra is rather tricky over $\mathbb{T}$

Thank you for your attention!

I hope that was of some help.