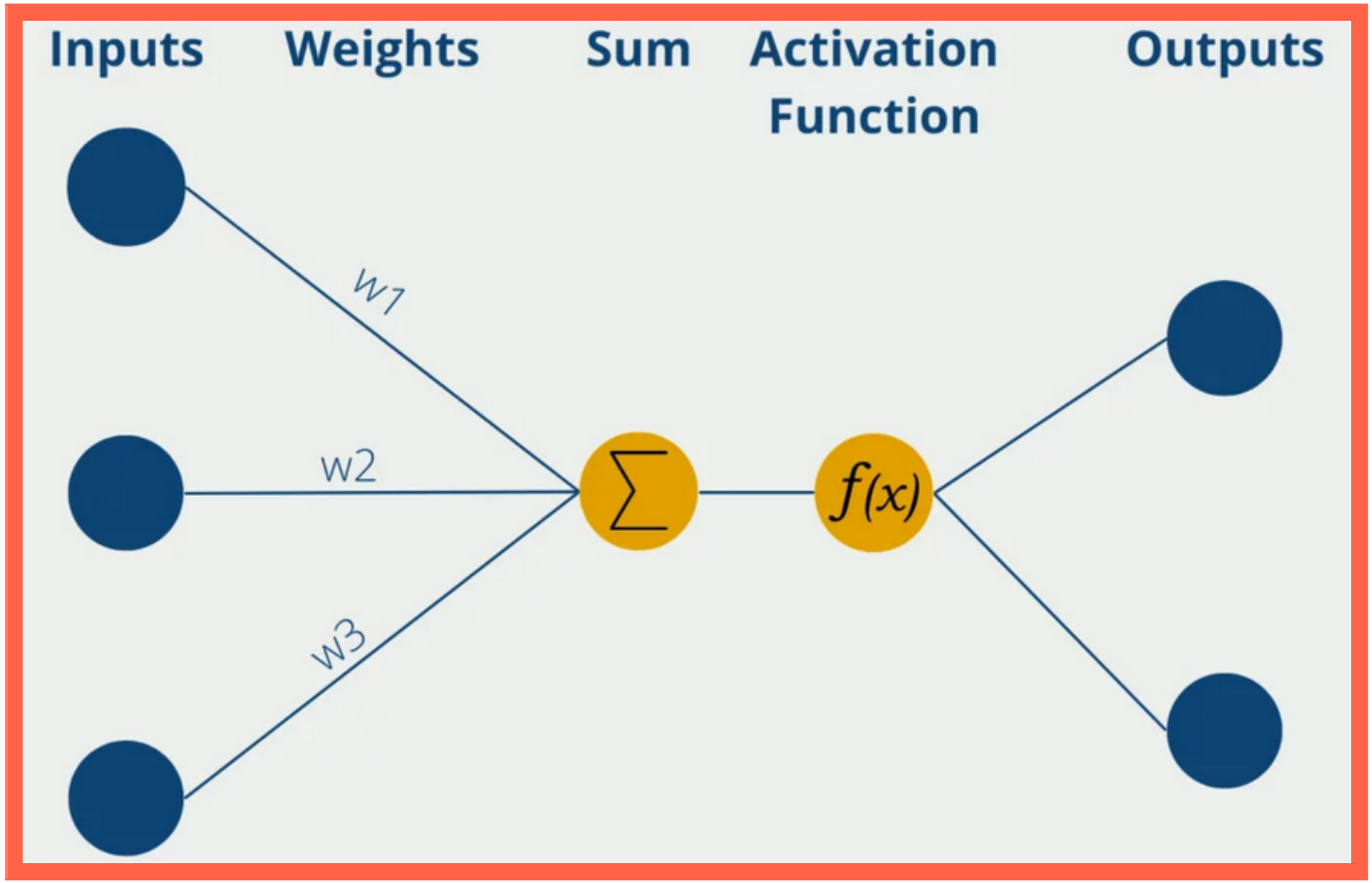
What is...tropical geometry - part 22?

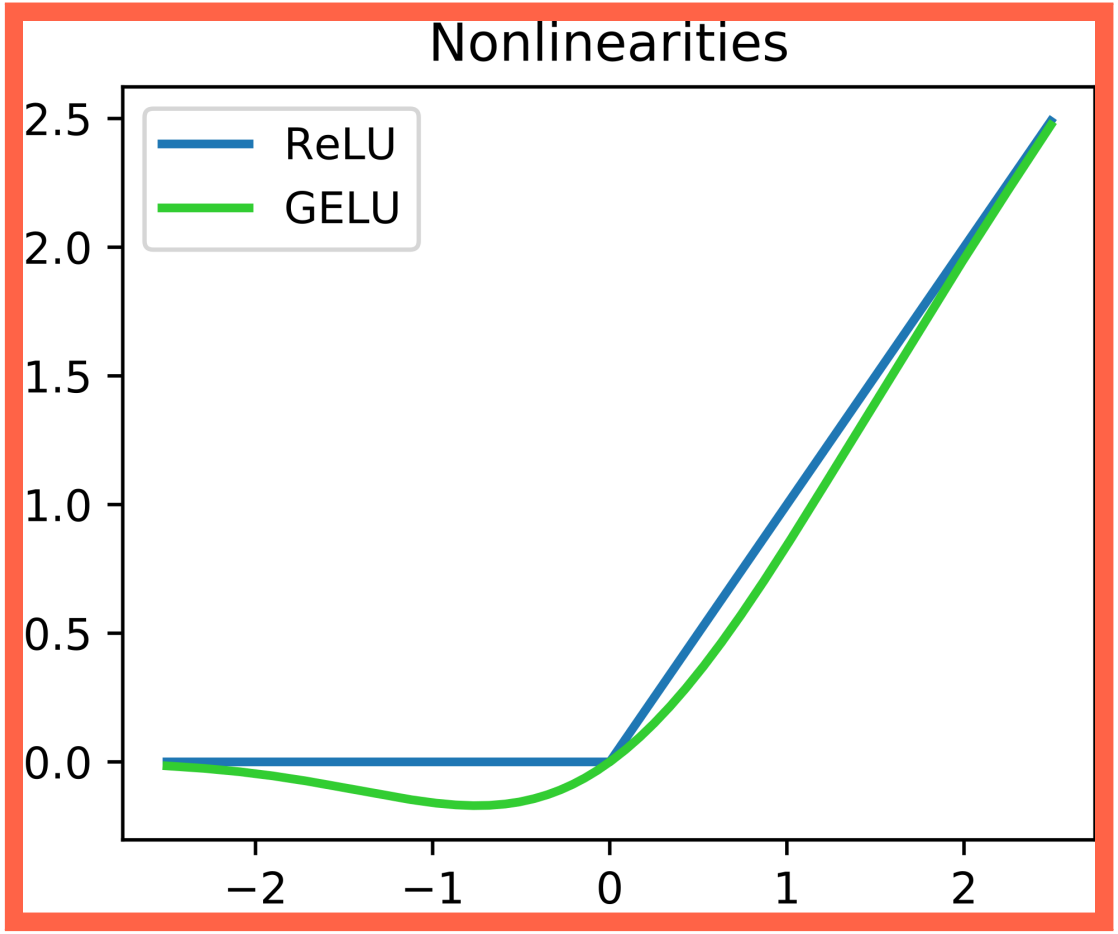
Or: Tropical applications 1 - The tropical geometry of neural nets

Neural nets (NN): a super brief reminder



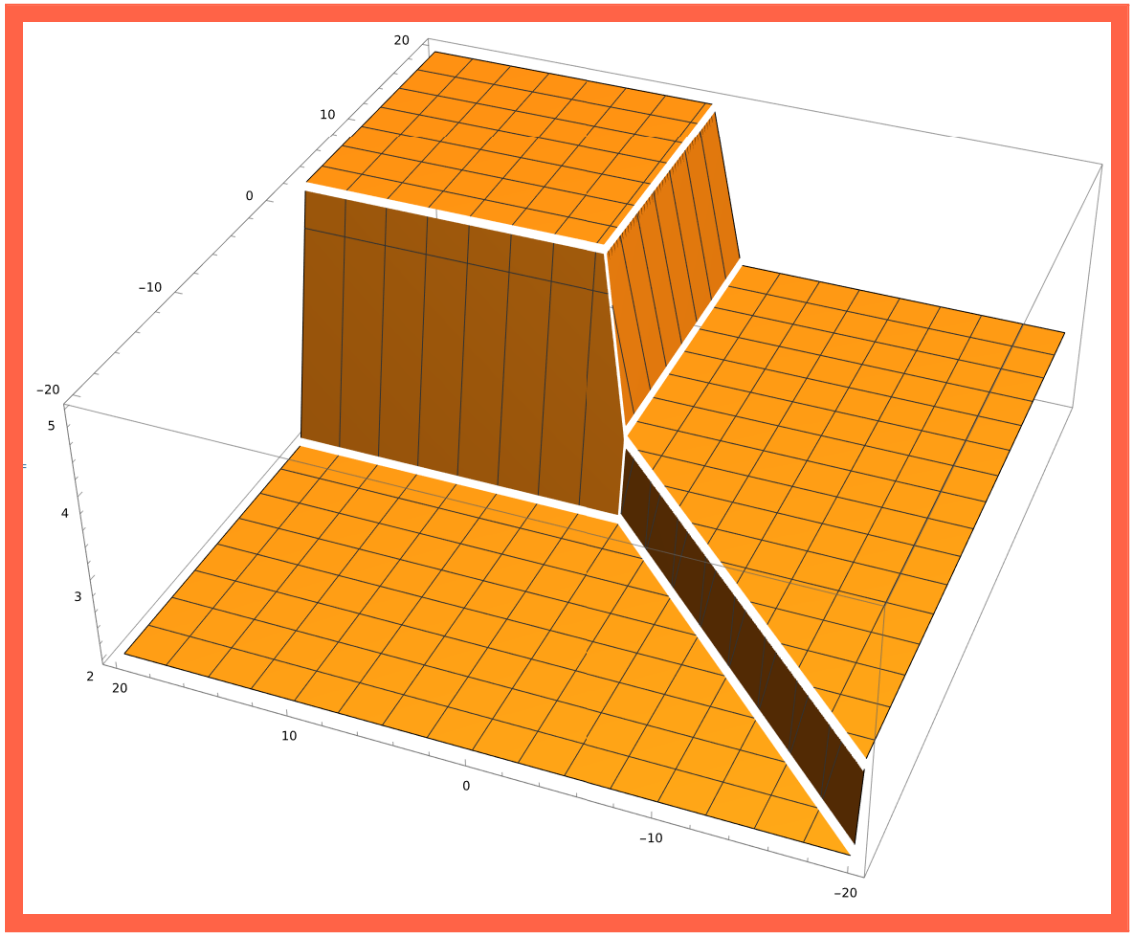
- NN = a function with Input-layers-output
- Layer = linear/affine map + ReLU gates
- Goal Recast this in tropical language (max-plus instead of min-plus)

Neural nets = piecewise linear geometry!?



- ReLU $= \max(0, x)$ $\Rightarrow$ piecewise linear layers
- Linear/Affine + ReLU $\Rightarrow$ polyhedral decision regions
- Tropical viewpoint: max-plus algebra captures this structure

Tropical rational functions (gentle version)



- Tropical arithmetic $a \oplus b = \max\{a, b\}, \quad a \odot b = a + b$
- Tropical polynomial $p(x) = \max_i\{a_i + \langle w_i, x\rangle\}$ max of affine forms)
- Tropical rational $f(x) = p(x) \ominus q(x)$ ($\rightsquigarrow f(x) = p(x) - q(x)$)

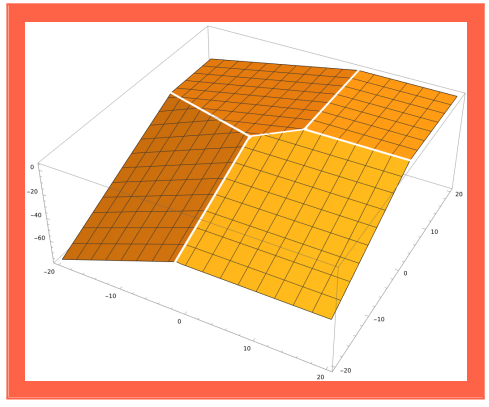
For completeness: A formal statement

We have:

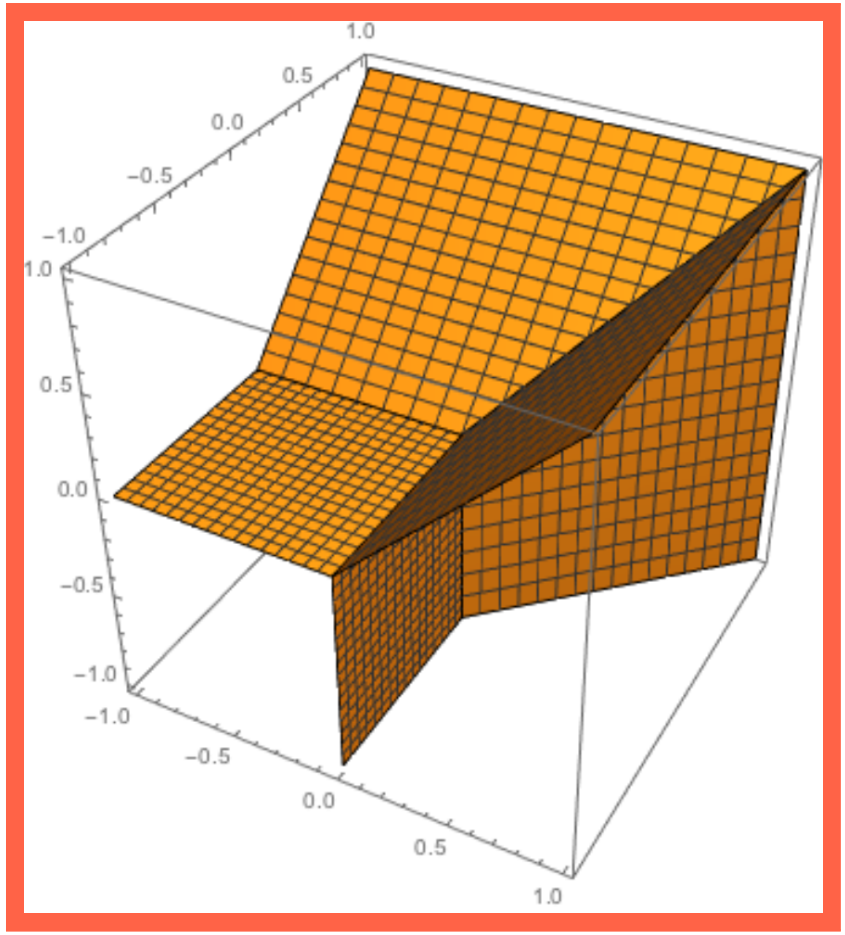
ReLU nets $\Leftrightarrow$ tropical rationals

meaning a ReLU network is a tropical rational function and vice versa

- Every feed-forward ReLU network computes a tropical rational $p \ominus q$
- Conversely, every tropical rational arises from some ReLU network
- Intuition Piecewise-linear functions are "often" differences of convex functions



Why this matters



- Interpretation ReLU NN are part of tropical geometry
- So we can use tropical geometry methods to study them
- NN questions can be translated in questions about tropical hypersurfaces etc.

Thank you for your attention!

I hope that was of some help.