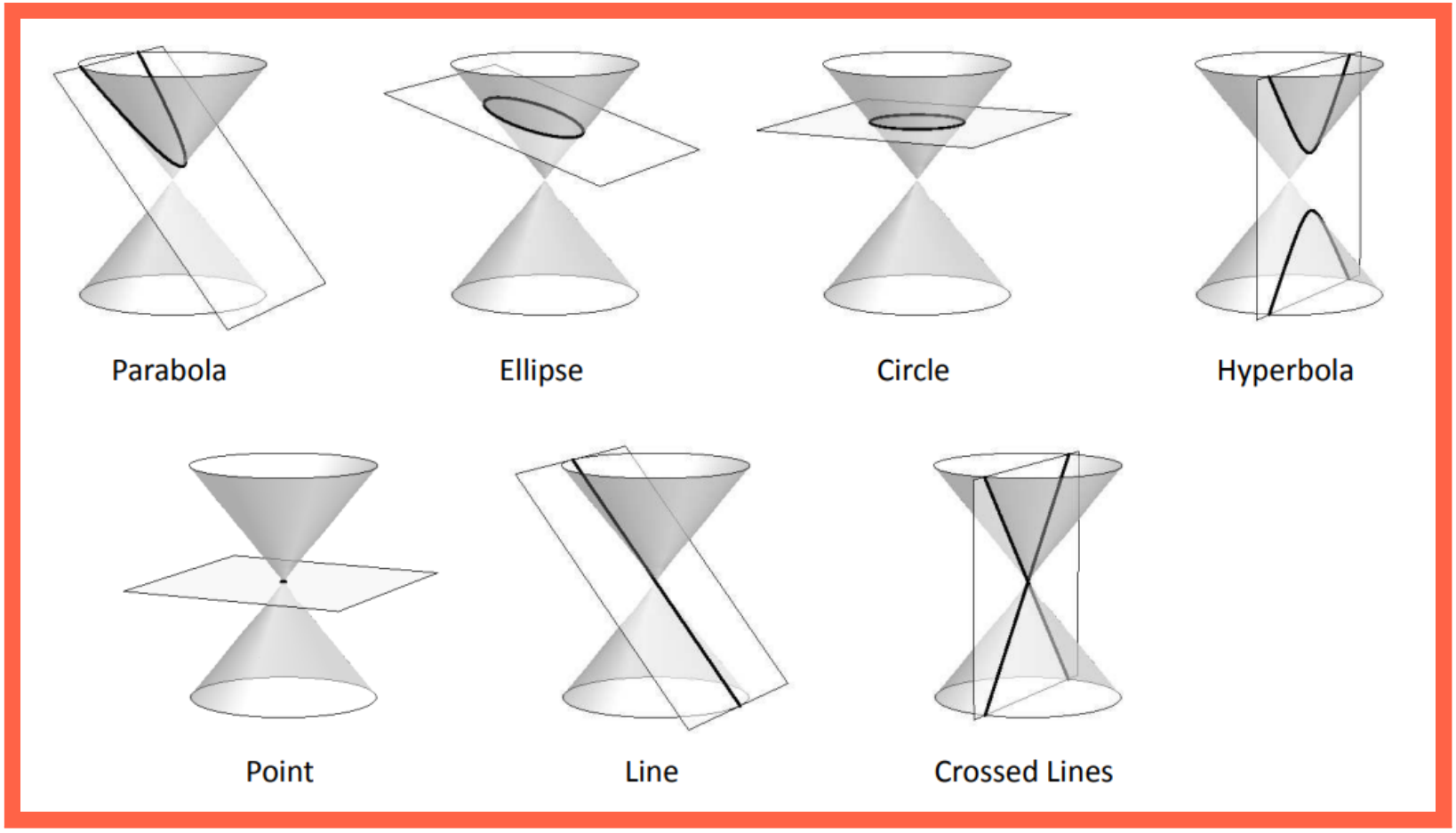
What is...tropical geometry - part 6?

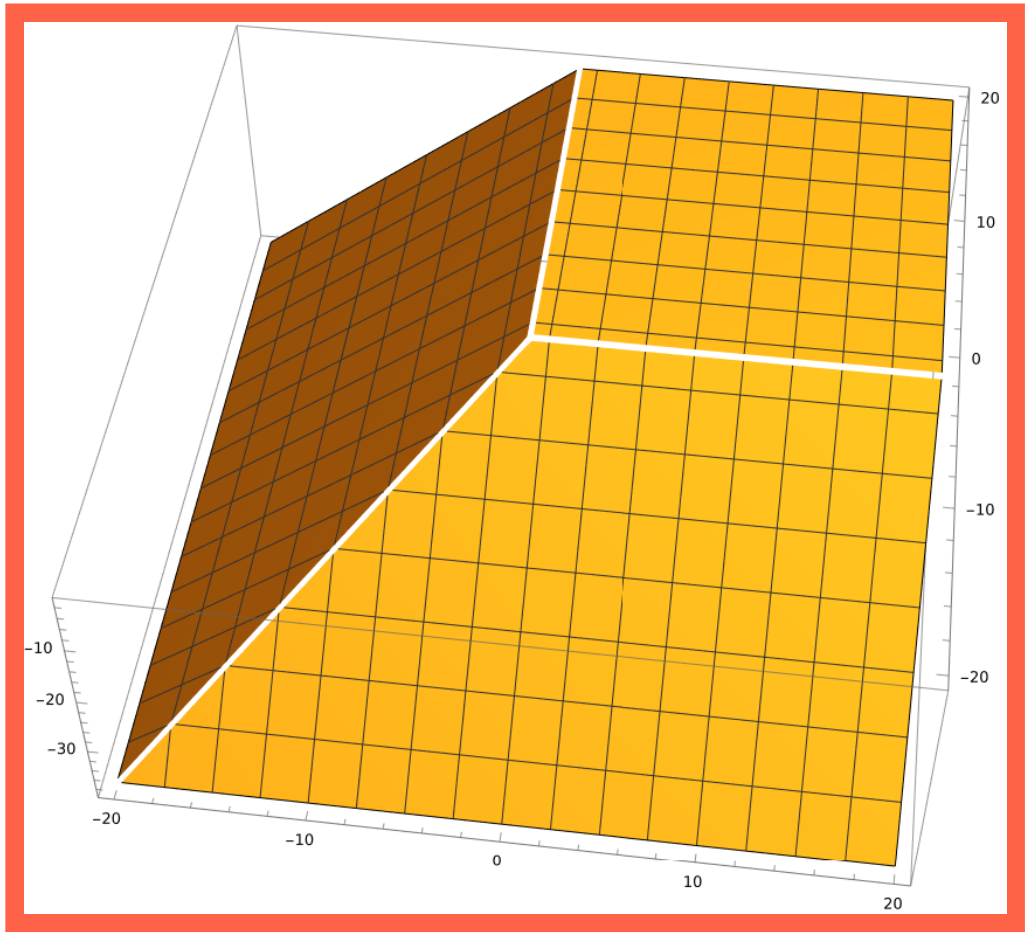
Or: Tropical varieties

Algebraic varieties



- Algebraic varieties = roots of polynomials
- Think: cut the function defined by the polynomial with a hyperplane
- Algebraic geometry is about algebraic varieties

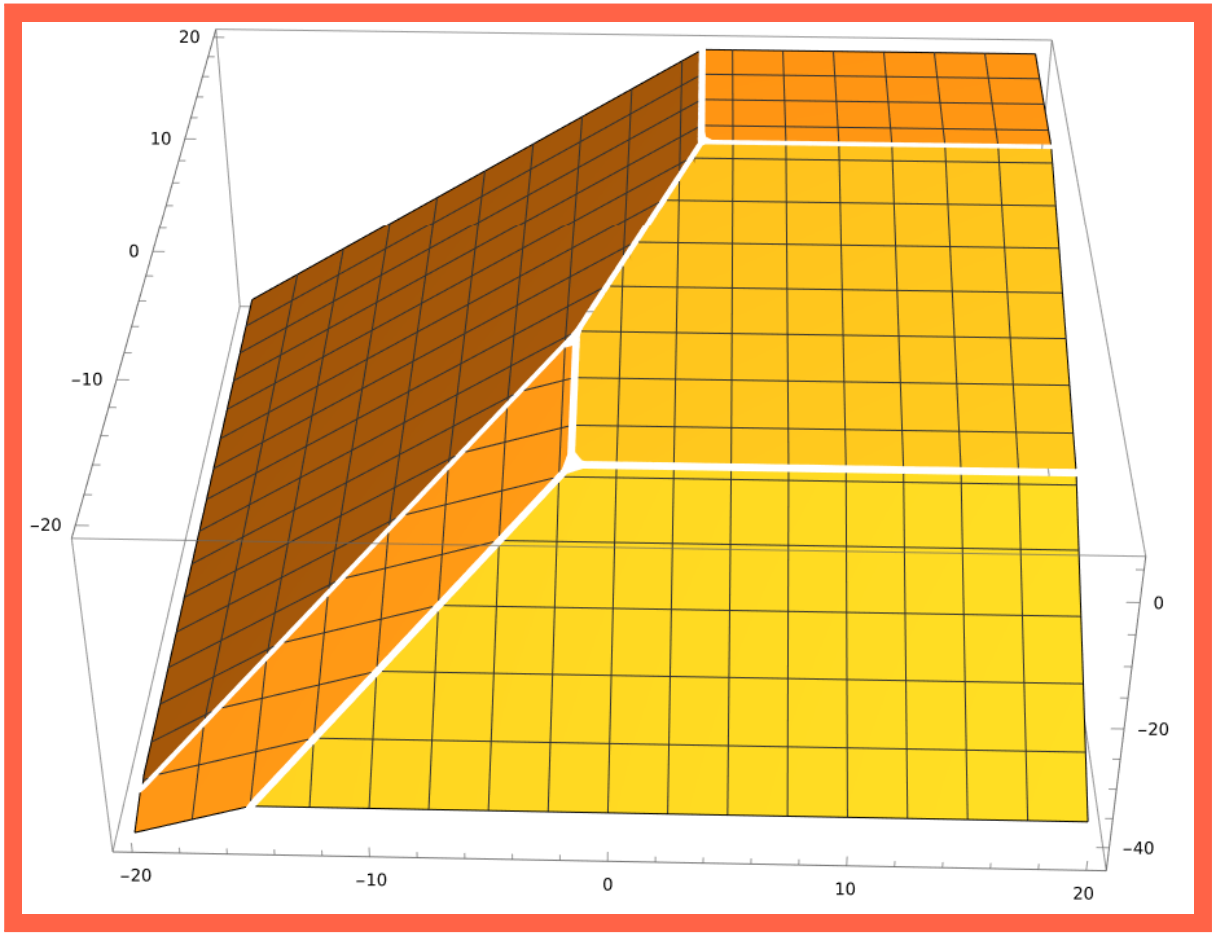
Tropical varieties



- Tropical varieties = roots of tropical polynomials
- Problem Cutting with hyperplanes gives 'wrong' results ⇒ not a good definition
- Question What is a tropical root?

Another tropical variety

$A =$



- Key The interesting parts of piecewise linear functions are its breaking points
- Tropical roots = the breaking points of the functions defined by tropical polynomials
- Example The white space above is a tropical variety

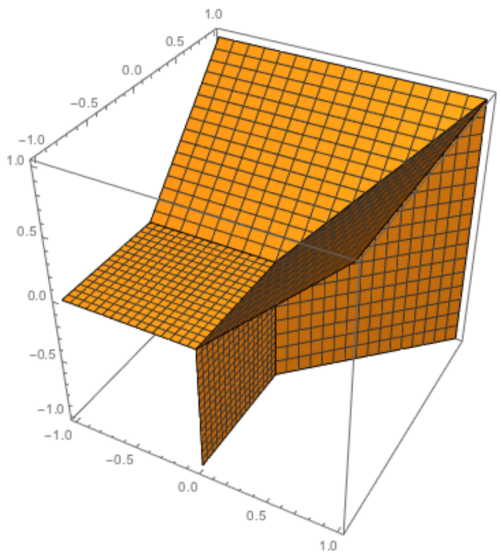
For completeness: A formal statement

Tropical polynomials function $= f : \mathbb{R}^n \to \mathbb{R}$ for a tropical polynomial f

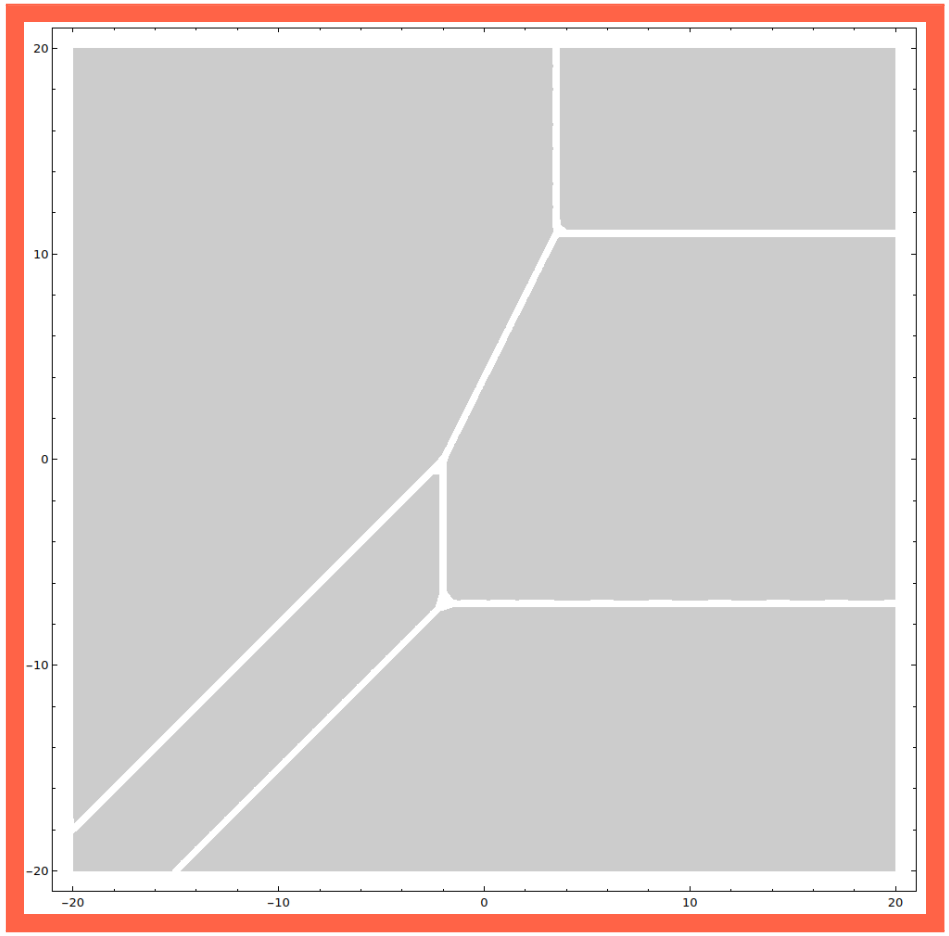
$$V(f) = \{x \in \mathbb{R}^n | f \text{ is not linear at } x\}$$

$V = V(f)$ is the tropical variety associated with f

- Warning You often see "minimum is attained at least twice" in the definition of V – that is equivalent
- Later we will see a more general definition
- An example for $n = 3$ is:



Contour plots (the tropical root graphs)



- Idea We like to see the tropical variety independent of the function
- To this end, we press the variety flat
- Example The tropical variety above is obtained from A

Thank you for your attention!

I hope that was of some help.