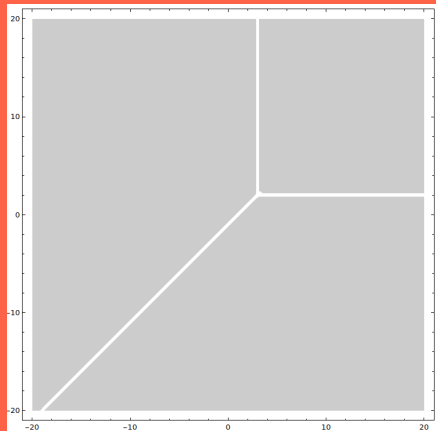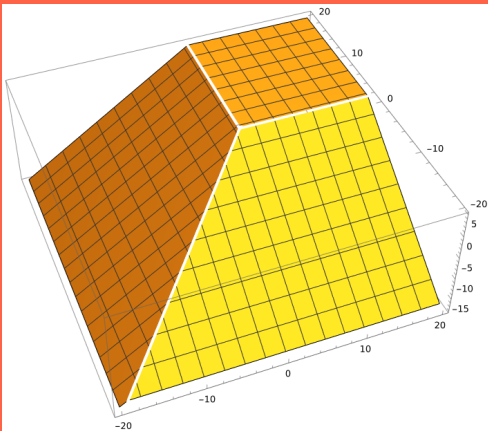**What is...tropical geometry - part 7?**
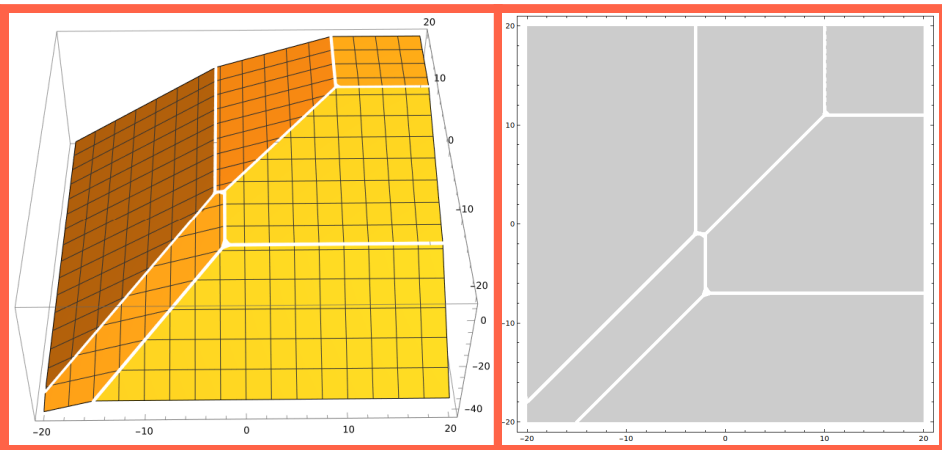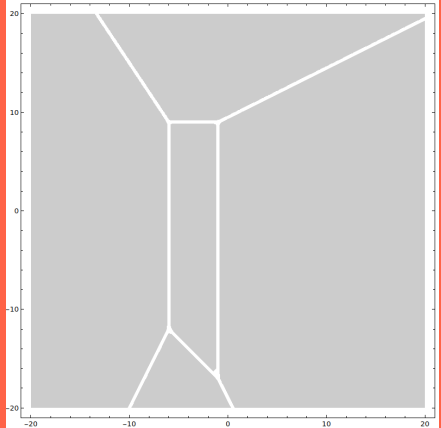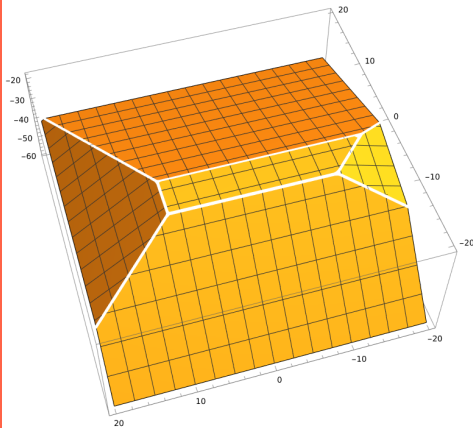
Or: Tropical curve

Tropical lines

- ▶ **Classical line** = solutions of $ax + by + c = 0$

- ▶ **Tropical line** = tropical solutions (breaking points) of $\min\{a + x, b + y, c\}$

- ▶ **Generic feature** Three components (corresponding to $a + x$, $b + y$, c)

Tropical quadrics

- ▶ Classical quadric = solutions of $ax^2 + by^2 + cxy + dx + ey + f = 0$

- ▶ Tropical quadric = tropical solutions of $\min\{a + 2x, b + 2y, c + x + y, d + x, e + y, f\}$

- ▶ Generic feature Six components (corresponding to $a + 2x, b + 2y, ...$)

Tropical elliptic curves

- ▶ Classical elliptic curve "=" smooth degree 3 curve of genus one

- ▶ Tropical elliptic curve = smooth degree 3 tropical curve of genus one

- ▶ Well let us figure out the definition of smooth degree 3 curve of genus one
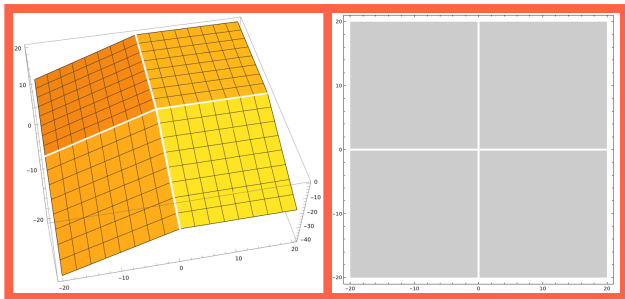
For completeness: A formal statement

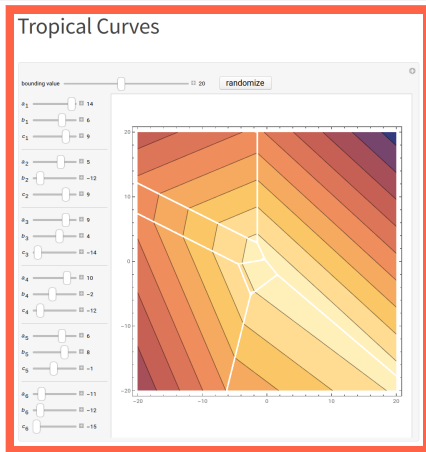Tropical polynomials function in two variables $= f \colon \mathbb{R}^2 \to \mathbb{R}$ for a tropical polynomial f

$$V(f) = \{x \in \mathbb{R}^n | f \text{ is not linear at } x\}$$

$V = V(f)$ is the tropical curve associated with f

▶ Terminology Degree = the exponents that appear (when reading as polynomials), smooth = only trivalent vertices, genus "=" number of holes

▶ A non-smooth example is:

More tropical fun

- ▶ Above A smooth tropical curve of genus one

- ▶ Beautiful internal edges and rays appear

- ▶ Play at https://demonstrations.wolfram.com/TropicalCurves/

Thank you for your attention!

I hope that was of some help.