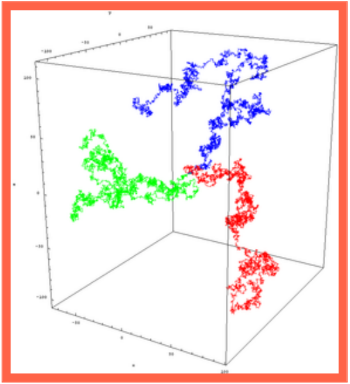
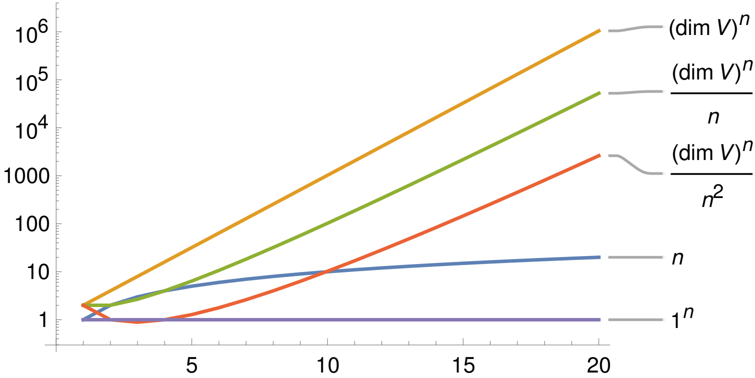
Analytic theory of monoidal categories

Or: Strategies to avoid counting



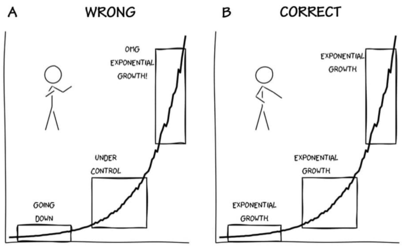
This is part 5

Avoiding counting in general



- Reminder We analyzed $b_n = \#$ inde. summands of $V^{\otimes n}$ for groups
- Question What about general categories?
- Additive Krull–Schmidt are assumed to hold together with $\mathbb{K}$-linear

Avoiding counting in general



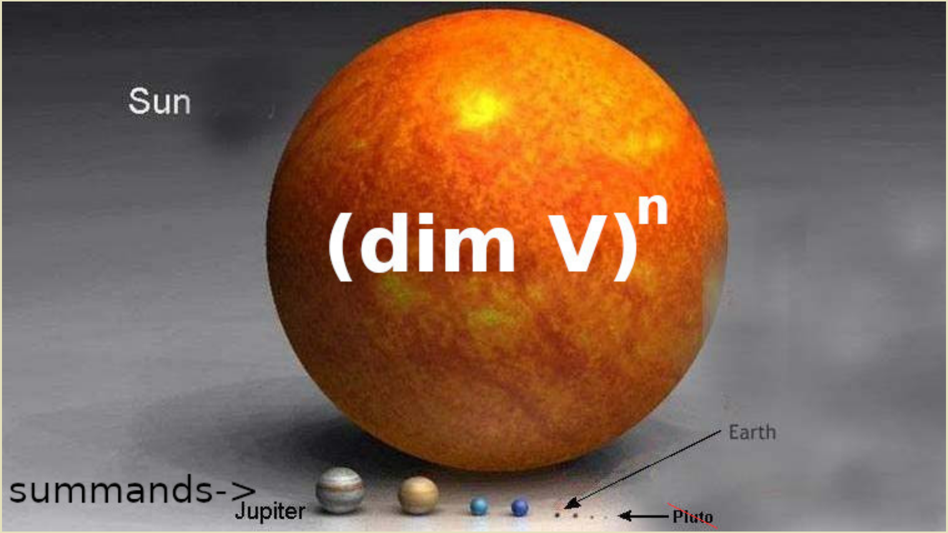
We have

$$\beta = \lim_{n\to\infty} \sqrt[n]{b_n} = \dim_{\mathbb{K}} V$$

- Start Recall that dominating growth is the dimension
- Question In what generality is that true?
- Minimal assumptions A functor $\mathcal{F}$ to vector spaces to define $\beta = \dim_{\mathbb{K}} \mathcal{F}(X)$

Exponential growth is scary

The "beta theorem $\beta = \dim_{\mathbb{K}} \mathcal{F}(X)$" holds if
X is an object in a symmetric monoidal cat
with a faithful symmetric functor $\mathcal{F}$ to vector spaces



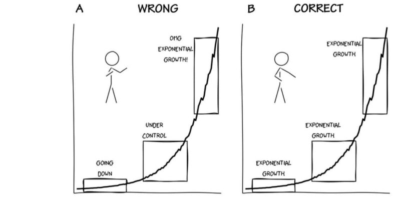
Avoiding counti

Examples

The "beta theorem $\beta = \dim_{\mathbb{K}} \mathcal{F}(X)$" applies to:

Representations of groups, monoids and semigroups

Superversions of these



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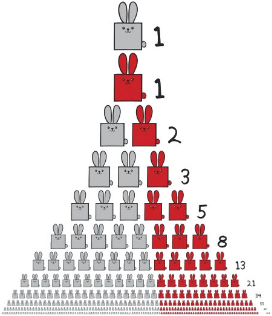
- Start Recall that dominating growth is the dimension
- Question In wha
- Minimal assumpt ... define $\beta = \dim_{\mathbb{K}} \mathcal{F}(X)$

What about more general categories?

For example, the Hecke category

Avoiding counting in general



- Rabbit counting à la Fibonacci using the matrices:

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, M^2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, M^3 = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, M^4 = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, \dots$$

- Thus, the growth rate of the entries of M^n is $\sim \sqrt{5} \cdot 1 = n^0 \cdot \phi^n$

The golden ratio ϕ is not an integer

Thus, for a category with $X^{\otimes 2} \cong X + 1$
the dominating growth of $b_n(X)$ is not a dimension of a space

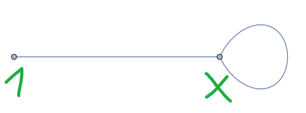
New methods are needed

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, M^2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, M^3 = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}, M^4 = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, \dots$$

b_1 b_2 b_3 b_4

Fusion graph/matrix M

Vertices = inde. objects, $Y \to Z$ if Z appears in $X \otimes Y$
No orientation = means $Y \to Z$ and $Y \leftarrow Z$



b_n = sum of unit column of M^n
b_n = number of paths of length n starting at the unit

- Rabbit c

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b_1 b_2 b_3 b_4

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Example

There is a category with $X^{\otimes 2} \cong X + 1$
and we have seen it:

Rep$(\mathbb{Z}/5\mathbb{Z}, \bar{\mathbb{F}}_5)$/projectives

$\mathbb{Z}/5\mathbb{Z}$ over $\bar{\mathbb{F}}_5$:

$$Z_1 = L_1 : \begin{pmatrix} 1 \end{pmatrix}, \text{ is simple}$$

$$Z_2 : \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \text{ filtration } 0 - L_1 - Z_2,$$

$$Z_3 : \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \text{ filtration } 0 - L_1 - L_1 - Z_3,$$

$$Z_4 : \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \text{ filtration } 0 - L_1 - L_1 - L_1 - Z_4,$$

$$Z_5 = P_1 : \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}, \text{ filtration } 0 - L_1 - L_1 - L_1 - L_1 - Z_5,$$

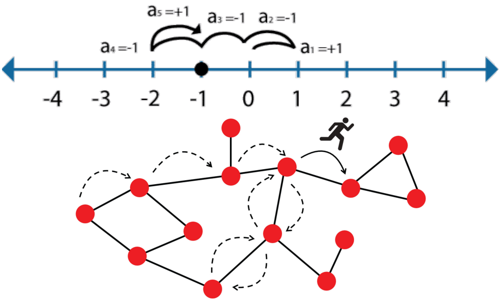
A

- Rabbit counting

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \dots M^4 = \begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix}, \dots$$

- Thus, the growth ... $1 = n^0 \cdot \phi^n$

Random walks and growth problems

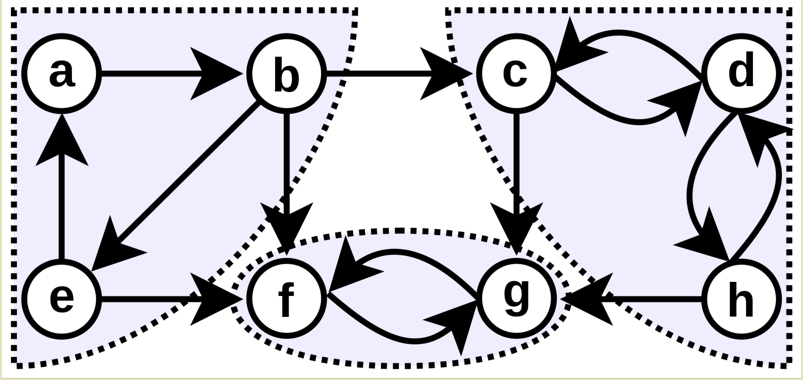


- We randomly walk on some (connected) graph = at each step choose the next step/edge randomly but equally likely "coin flip walk"
- Question How often do we visit a vertex?
- Recurrent := We will hit the start infinitely often with P(rob)=1 ⇔ one finds home; transient := not recurrent ⇔ one moves out

Example

Random walks on (strongly connected) finite graphs are recurrent

From now: graphs are strongly connected



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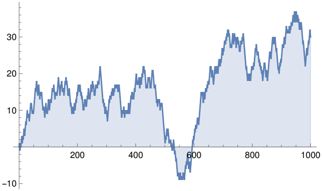
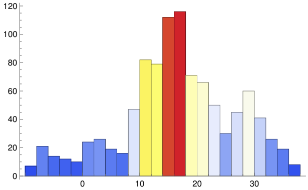
Random walks and growth problems

A random walk:



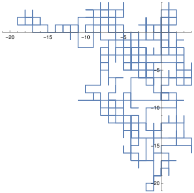
The distribution of how far one is from home:



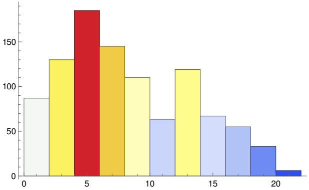
- **1d random walk** = take a step left / right with probability 1/2
- **Question** What is the probability p_{home} of return to the origin (=home)?
- **Plotting this** convinces one quickly that $p_{home} = 1$

Random walks and growth problems

A random walk:



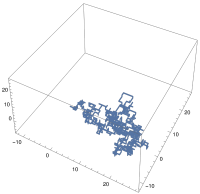
The distribution of how far one is from home :



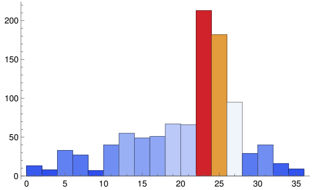
- 2d random walk = take a step left / right / up / down with probability 1/4
- Question What is the probability p_{home} of return to the origin (=home)?
- Plotting this gives a quite ambiguous result

Random walks and growth problems

A random walk:

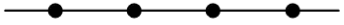


The distribution of how far one is from home:

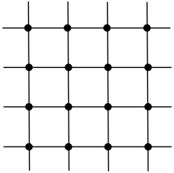


- 3d random walk = take a step left / right / up / down / in / out with probability 1/6
- Question What is the probability p_{home} of return to the origin (=home)?
- Plotting this convinces one quickly that $p_{home} < 1$

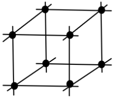
Random walks and growth problems



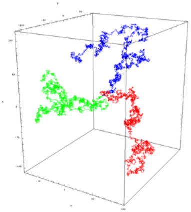
1-dimensional lattice



2-dimensional lattice



3-dimensional lattice



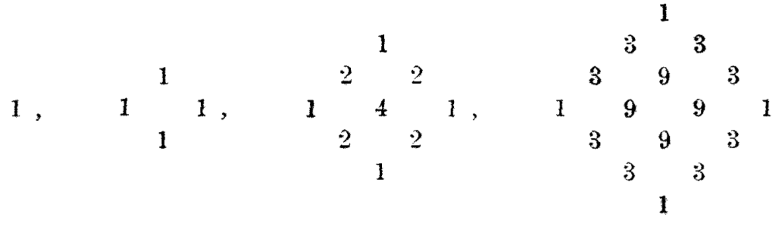
- **Theorem (Pólya ∼1921)** $\mathbb{Z}^d$ is recurrent/transient $\Leftrightarrow$ $d \leq 2/d > 2$
- A drunkard will find their way home, but a drunken bird may get lost forever
- Observation Extending $\mathbb{Z}^d$ by a finite graph does not change the theorem

Pólya ∼1921

Über eine Aufgabe der Wahrscheinlichkeitsrechnung betreffend die Irrfahrt im Straßennetz.

Von

Georg Pólya in Zürich.



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Random walks and growth problems

Call b_n recurrent/transient if its fusion graph is

Fusion graph/matrix M

Vertices = inde. objects, $Y \to Z$ if Z appears in $X \otimes Y$
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b_n = sum of unit column of M^n
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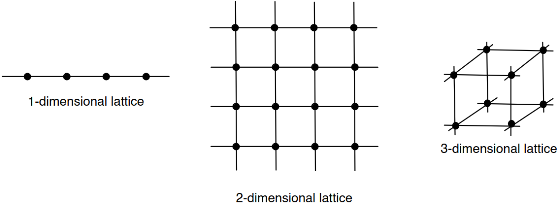
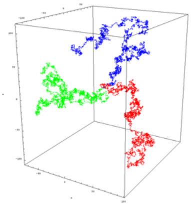
Random walks and growth problems





- **Theorem (Pólya $\sim$1921)** $b_n(V)$ for V a completely reducible faithful group rep in char zero is recurrent $\Leftrightarrow$ Γ is virtually $\mathbb{Z}^d$ for $d \in \{0, 1, 2\}$
- Virtually means we allow extensions by finite groups

Rand

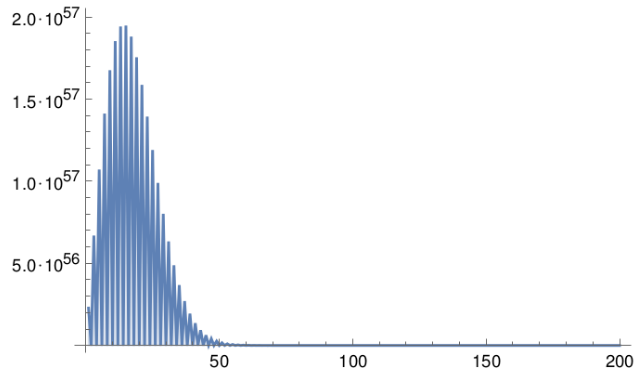
Example

The fusion graph of $SL_2(\mathbb{C})$ and $V = \mathbb{C}^2$ is $\mathbb{N}$

$$\Gamma(\mathrm{SL}_2) = \bullet \rightleftarrows \bullet \rightleftarrows \bullet \rightleftarrows \bullet \rightleftarrows \bullet \rightleftarrows \bullet \rightleftarrows \cdots,$$

$$M(\mathrm{SL}_2) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \dots \\ 1 & 0 & 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & 0 & 1 & \dots \\ 0 & 0 & 0 & 1 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}.$$

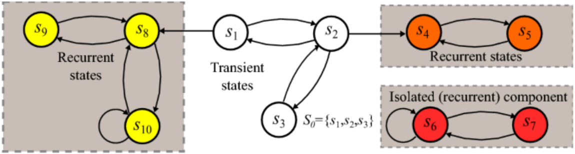
This is transient: the number of paths moves out; here (end vertex,#paths)



▶ ... group r...

▶ Virtually means we allow extensions by finite groups

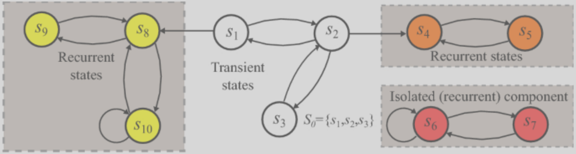
Random walks and growth problems



For what Γ and V is b_n recurrent/transient?

- Recurrent
  - (i) Arbitrary field, finite group Γ and V any Γ-rep
  - (ii) Arbitrary field, finite tensor cat Γ and V any object
  - (iii) Arbitrary field, Hecke cat Γ for a finite Coxeter group and V any object
- Transient
  - (i) Char. zero, any group such that $\Gamma \subset \mathrm{GL}(V)$ is not a torus of rank $0, 1, 2$
  - (ii) Some assumptions, quantum group Γ and V any nontrivial tilting rep
  - (iii) Char. zero, Hecke cat Γ for an affine Weyl group and V any nontrivial object

Random walks and growth problems



For what Γ and V is

Conjecture

"Everything" not on this list is transient

- Recurrent
  - (i) Arbitrary field, finite group Γ and V any Γ-rep
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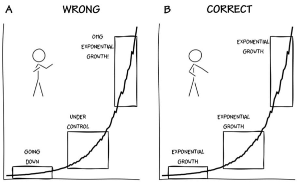
Asymptotics for recurrent categories



We have

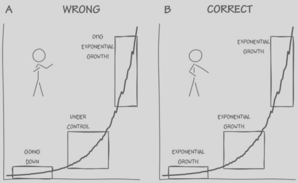
$$\beta = \lim_{n\to\infty} \sqrt[n]{b_n} = \dim_{\mathbb{K}} V$$

- **Theorem** For certain recurrent problems the beta theorem is true with $\beta =$ largest eigenvalue of M and

$$b_n \sim a_n = h(n) \cdot n^0 \cdot \beta^n$$

where $h\colon \mathbb{N} \to (0,1]$ is periodic of finite period (we do this in a second)

- **certain** $=$ some annoying assumptions that are omitted



Example

All recurrent problems we have seen so far are certain

- Theorem For certain recurrent problems the beta theorem is true with $\beta =$ largest eigenvalue of M and

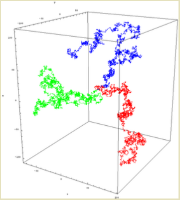
$$b_n \sim a_n = h(n) \cdot n^0 \cdot \beta^n$$

where $h \colon \mathbb{N} \to (0, 1]$ is periodic of finite period (we do this in a second)

- certain = some annoying assumptions that are omitted

On the next slide there is a formula of the form

$$\underbrace{b_n}_{b(n)} \sim \underbrace{c(n) \cdot (\dim_{\mathbb{K}} V)^n}_{a(n)} \text{ ''}$$



We will explore the formula by examples
so no need to memorize it

The take away messages are:

The formula is completely explicit and works in quite some generality specified later

It only depends on eigenvalues and eigenvectors associated to a matrix

The assumptions on the next slide are not necessary
but make the formula look nicer

Asymptotics for recurrent categories

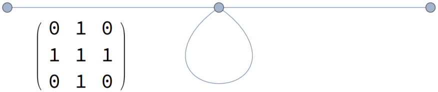
- Take a finite based $\mathbb{R}_{\geq 0}$-algebra R with basis $C = \{c_0, ..., c_{r-1}, ...\}$
- Assume that R is the Grothendieck ring of our starting category
- For $a_i \in \mathbb{R}_{\geq 0}$, the action matrix M of $c = a_0 \cdot c_0 + ... + a_{r-1} \cdot c_{r-1} \in R$ is the matrix of left multiplication of c on C
- Assume that M has a leading eigenvalue λ of multiplicity one; all other eigenvalues of the same absolute value are $\exp(k2\pi i/h)\lambda$ for some h
- Denote the right and left eigenvectors of M for λ and $\exp(k2\pi i/h)\lambda$ by v_i and w_i, normalized such that $w_i^T v_i = 1$
- Let $v_i w_i^T[1]$ denote taking the sum of the first column of the matrix $v_i w_i^T$
- The formula $b(n) \sim a(n)$ we are looking for is ($\zeta = \exp(2\pi i/h)$)

$$b(n) \sim \big(v_0 w_0^T[1] \cdot 1 + v_1 w_1^T[1] \cdot \zeta^n + v_2 w_2^T[1] \cdot (\zeta^2)^n + ... + v_{h-1} w_{h-1}^T[1] \cdot (\zeta^{h-1})^n\big) \cdot \lambda^n$$

- The variance is $|b_n - a_n| \leq (\lambda_{sec})^n + n^d$

Asymptotics for recurrent categories

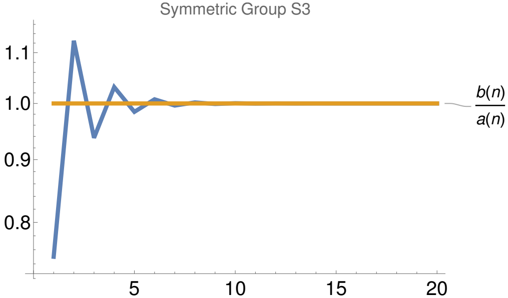
Symmetric group S_3, $\mathbb{K} = \mathbb{C}$, V=standard rep



$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Example $\lambda = 2$, others$=0, -1$, $v = w = 1/\sqrt{6}(1,2,1)$, $vw^T = \begin{pmatrix} 1/6 & 1/3 & 1/6 \\ 1/3 & 2/3 & 1/3 \\ 1/6 & 1/3 & 1/6 \end{pmatrix}$ and

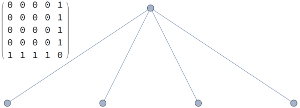
$$a(n) = \frac{2}{3} \cdot 2^n$$



Asymptotics for recurrent categories

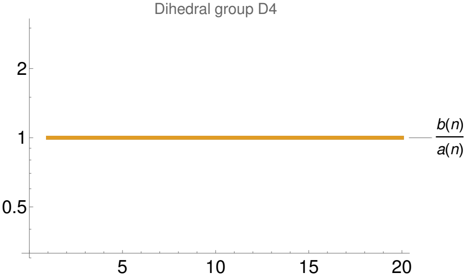
Dihedral group D_4 of order 8, $\mathbb{K} = \mathbb{C}$, V=defining rotation rep

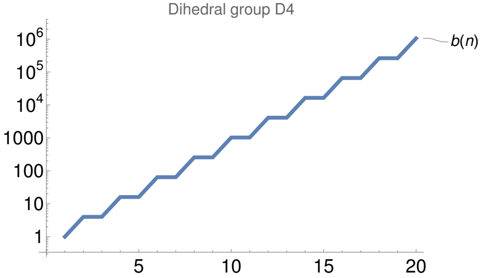
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$



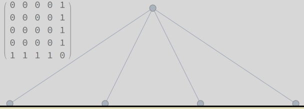
Example $\lambda = 2$, others$=-2, 0, 0, 0$, $v_\lambda = w_\lambda = 1/\sqrt{8}(1, 1, 1, 1, 2)$
$v_{-2} = w_{-2} = 1/\sqrt{8}(-1, -1, -1, -1, 2)$ and

$$a(n) = \left(\tfrac{3}{4} + \tfrac{1}{4}(-1)^n\right) \cdot 2^n$$





Dihedral group D_4 of order 8, $\mathbb{K}=\mathbb{C}$, V=defining rotation rep

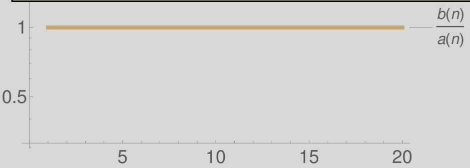


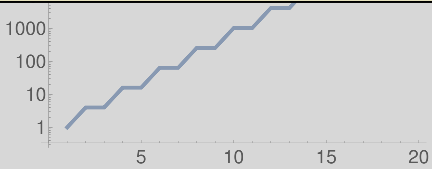
Example (general finite group, $\mathbb{K}=\mathbb{C}$, V=any faithful G-rep)

In this case we have a general formula:

$$a(n)=\left(\frac{1}{\#G}\sum_{g\in Z_V(G)}\Big(\sum_{L\in \mathrm{S}(G)}\omega_L(g)\dim_{\mathbb{C}}L\Big)\cdot\omega_V(g)^n\right)\cdot(\dim_{\mathbb{C}}V)^n$$

$Z_v(G)$=elements g acting by a scalar $w_V(g)$; $S(G)$=set of simples





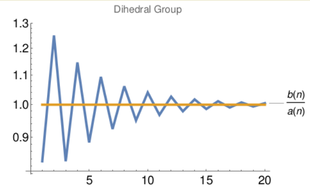
Example (continued)

Symmetric group S_m $a(n) = \left(\sum_{k=0}^{m/2} 1/((m-2k)!k!2^k)\right) \cdot (\dim_{\mathbb{C}} V)^n$

Dihedral group D_m of order $2m$

$$a(n) = \begin{cases} \frac{m+1}{2m} \cdot 2^n & \text{if } m \text{ is odd,} \\ \frac{m+2}{2m} \cdot 2^n & \text{if } m \text{ is even and } m' \text{ is odd,} \\ \left(\frac{(m+2)}{2m} \cdot 1 + \frac{1}{m} \cdot (-1)^n\right) \cdot 2^n & \text{if } m \text{ is even and } m' \text{ is even.} \end{cases}$$

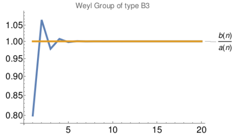
$m = 5$:



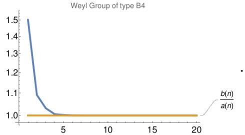
Complex reflection group $G(d, 1, m)$

$$\begin{cases} d = 1, \\ m = 3 \end{cases} : a(n) = \frac{2}{3} \cdot 3^n, \quad \begin{cases} d = 2, \\ m = 3 \end{cases} : a(n) = \frac{5}{12} \cdot 3^n, \quad \begin{cases} d = 2, \\ m = 4 \end{cases} : a(n) = \left(\frac{19}{96} \cdot 1 + \frac{1}{32} \cdot (-1)^n\right) \cdot 4^n$$

$\begin{cases} d = 2, \\ m = 3 \end{cases}$:



, $\begin{cases} d = 2, \\ m = 4 \end{cases}$:



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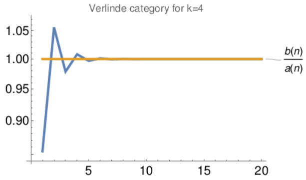
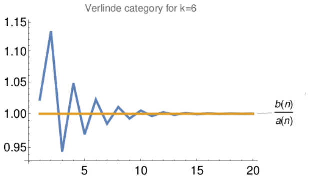
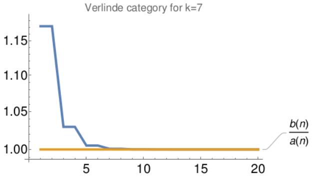
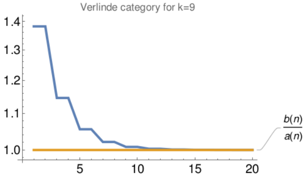
Asymptotics for recurrent categories

$k=4:$ 

, $k=6:$ 

$k=7:$ 

, $k=9:$ 

Example For the SL_2 Verlinde category over $\mathbb{C}$ at level *k* and *V*=gen. object:

$$a(n)=\begin{cases}\frac{[1]_q+\dots+[k]_q}{[1]_q^2+\dots+[k]_q^2}\cdot\big(2\cos(\pi/(k+1))\big)^n & \text{if } k \text{ is even,}\\ \Big(\frac{[1]_q+\dots+[k]_q}{[1]_q^2+\dots+[k]_q^2}\cdot 1+\frac{[1]_q-[2]_q+\dots-[k-1]_q+[k]_q}{[1]_q^2+\dots+[k]_q^2}\cdot(-1)^n\Big)\cdot\big(2\cos(\pi/(k+1))\big)^n & \text{if } k \text{ is odd.}\end{cases}$$

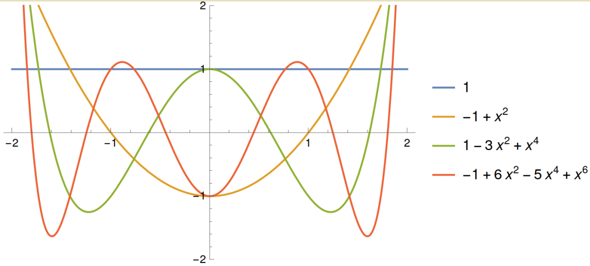
Asymptotics

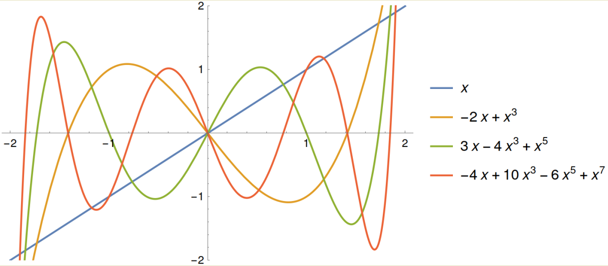
$k = 4:$

$k = 7:$

Example (continued)

The growth rate in this case is **not in $\mathbb{N}$**
but rather the leading root of the Chebyshev polynomial:





Example For … n. object:

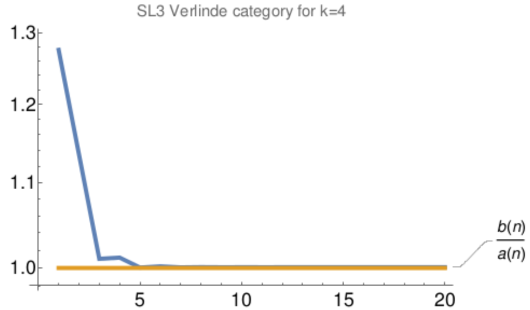
$a(n) = \begin{cases} \frac{[1]_q + \dots}{[1]_q^2 + \dots} & \text{if } k \text{ is even,} \\ \left(\frac{[1]_q + \dots}{[1]_q^2 + \dots}\right)^n & \text{if } k \text{ is odd.} \end{cases}$

Example (continued)

Here is the SL_3 Verlinde category over $\mathbb{C}$ at level $k = 4$ and V=gen. object:

$$k = 4\colon a(n) = \frac{1}{7}\left(2 + 2\cos\left(\frac{3\pi}{7}\right)\right) \cdot \left(1 + 2\cos\left(\frac{2\pi}{7}\right)\right)^{n},$$

$k = 4$:



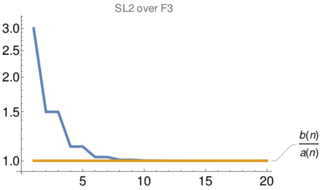
.

Koornwinder polynomials make their appearance

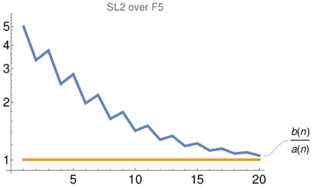
$\cdot\,(2\cos(\pi/(k+1)))$ if k is odd.

Asymptotics for recurrent categories

$p = 3:$



$p = 5:$

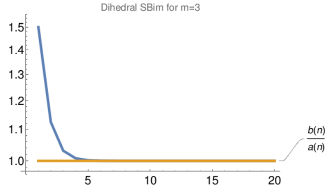


Example For $\mathrm{SL}_2(\mathbb{F}_p)$, $\mathbb{K} = \mathbb{F}_p$ and $V = \mathbb{F}_p^2$ we get:

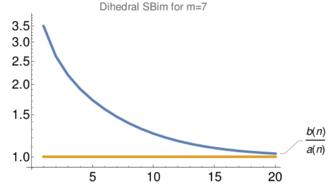
$$a(n) = \left(\frac{1}{2p-2} \cdot 1 + \frac{1}{2p^2 - 2p} \cdot (-1)^n \right) \cdot 2^n$$

Asymptotics for recurrent categories

$m = 3$:



$m = 7$:

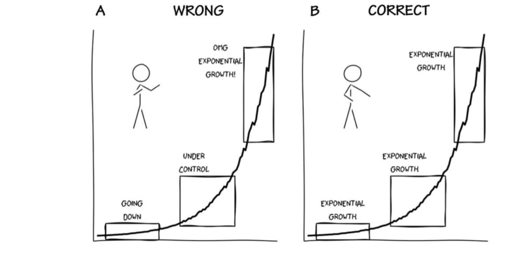


Example For dihedral Soergel bimodules of D_m, $\mathbb{K} = \mathbb{C}$ and $V = B_{st}$ we get:

$$a(n) = \frac{1}{2m} \cdot 4^n$$

Asymptotics for recurrent categories

Observe that the growth of $b(n)$ is always exponential



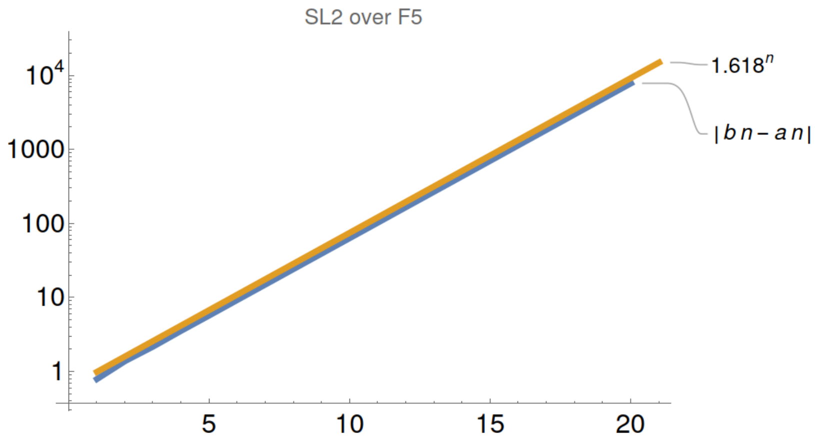
We have

$$\beta = \lim_{n\to\infty} \sqrt[n]{b_n} = \dim_{\mathbb{K}} V$$

Example For dihedral Soergel bimodules of D_m, $\mathbb{K} = \mathbb{C}$ and $V = B_{st}$ we get:

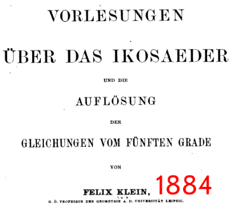
$$a(n) = \frac{1}{2m} \cdot 4^n$$

Asymptotics for recurrent categories



- The variance is given by $(\lambda_{sec})^n$ (second largest EV)
- Example Above for $\mathrm{SL}_2(\mathbb{F}_5)$, $\mathbb{K} = \mathbb{F}_5$ and $V = \mathbb{F}_5^2$, λ_{sec}=golden ratio

Asymptotics for recurrent categories


VORLESUNGEN

ÜBER DAS IKOSAEDER

UND DIE

AUFLÖSUNG

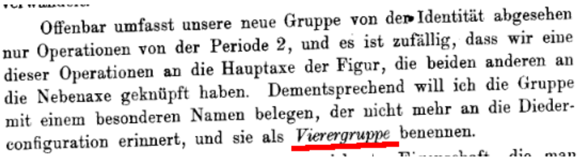
DER

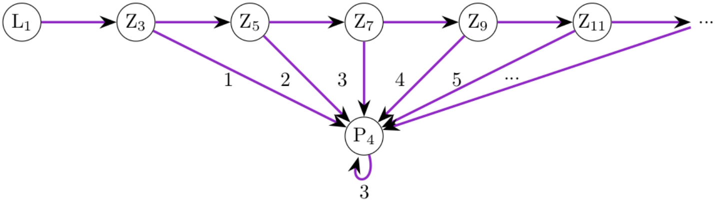
GLEICHUNGEN VOM FÜNFTEN GRADE

VON

FELIX KLEIN, 1884

O. Ö. PROFESSOR DER GEOMETRIE A. D. UNIVERSITÄT LEIPZIG.

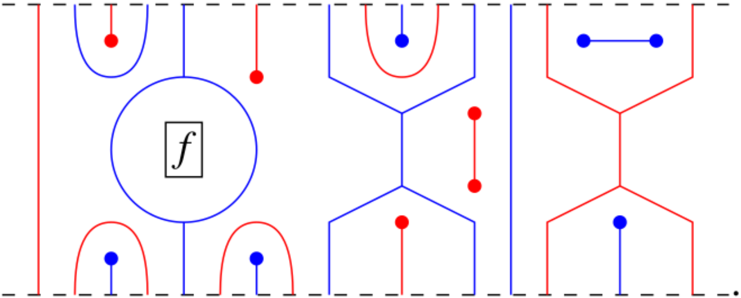

Offenbar umfasst unsere neue Gruppe von der Identität abgesehen nur Operationen von der Periode 2, und es ist zufällig, dass wir eine dieser Operationen an die Hauptaxe der Figur, die beiden anderen an die Nebenaxe geknüpft haben. Dementsprechend will ich die Gruppe mit einem besonderen Namen belegen, der nicht mehr an die Dieder-configuration erinnert, und sie als *Vierergruppe* benennen.



Example For the Klein four group $\mathbb{Z}/2\mathbb{Z}\times\mathbb{Z}/2\mathbb{Z}$, $\mathbb{K}=\overline{\mathbb{F}_2}$ and $V=Z_3$=3d inde. we get:

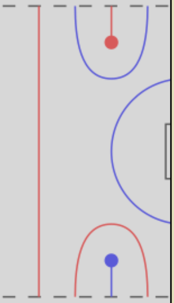
$$b_n \sim 3^n$$

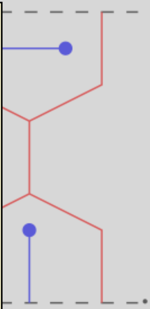
Asymptotics for recurrent categories



- The Hecke category (finally) for a finite Coxeter group W, $\mathbb{K}$ char zero field, V = any indecomposable object, $[V] = \sum_{s \in W} m_s \cdot s$
- We have

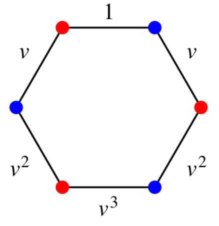
$$b_n \sim a_n = 1/|W| \cdot n^0 \cdot (\textstyle\sum_{s \in W} m_s)^n$$





Example

If V is inde., then the m_s are the Kazhdan–Lusztig polynomials eval. at 1
e.g. for S_3 and $V = B_{sts}$
we get $[V] = 1 + s + t + st + ts + sts$

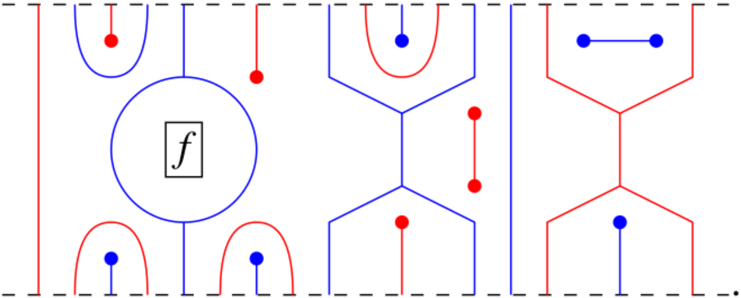


$b_n \sim \frac{1}{6} \cdot n^0 \cdot 6^n$

- The Hecke categ … W, $\mathbb{K}$ char zero field, V = any indecom …
- We have

$$b_n \sim a_n = 1/|W| \cdot n^0 \cdot (\textstyle\sum_{s \in W} m_s)^n$$

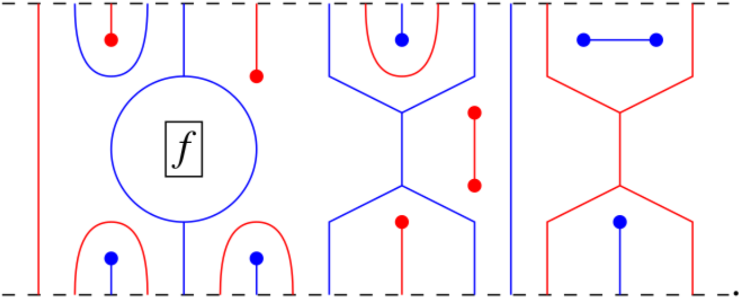
Asymptotics for recurrent categories



- The Hecke category (finally) for an affine Weyl group W, $\mathbb{K}$ char zero field, V = any indecomposable object, $[V] = \sum_{s \in W} m_s \cdot s$
- We have

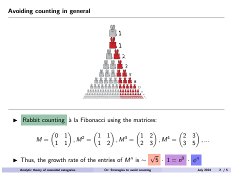
$$b_n \sim a_n = ??? \cdot n^{-\#\text{pos. roots}/2} \cdot \left(\sum_{s \in W} m_s\right)^n$$

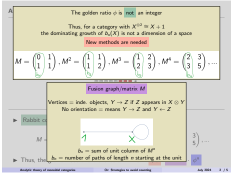
Asymptotics for recurrent categories

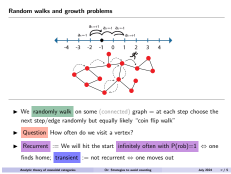


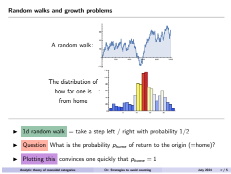
- The Hecke category (finally) for any other Coxeter group W, $\mathbb{K}$ char zero field, V = any indecomposable object, $[V]=\sum_{s\in W} m_s \cdot s$
- We have

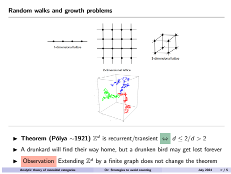
$$b_n \sim a_n = ??? \cdot n^{???} \cdot \Big(\sum_{s\in W} m_s\Big)^n$$

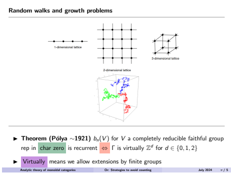


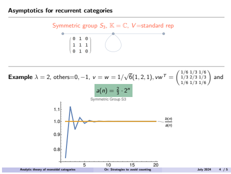


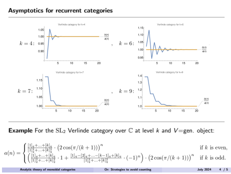


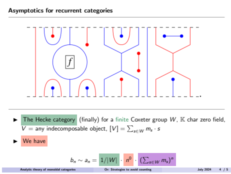




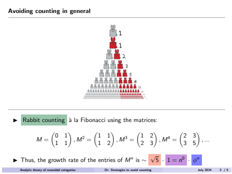


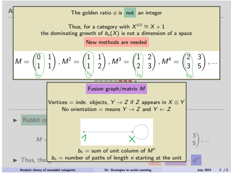


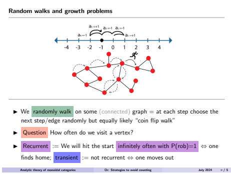


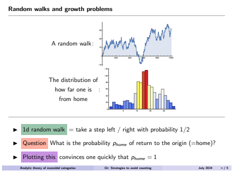


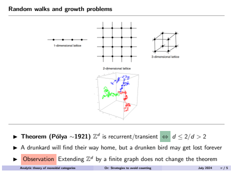
There is still much to do...

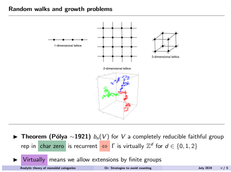


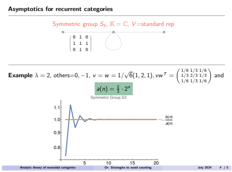


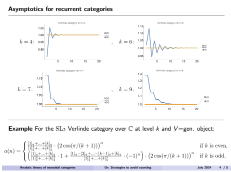


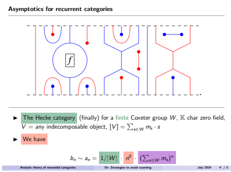












Thanks for your attention!