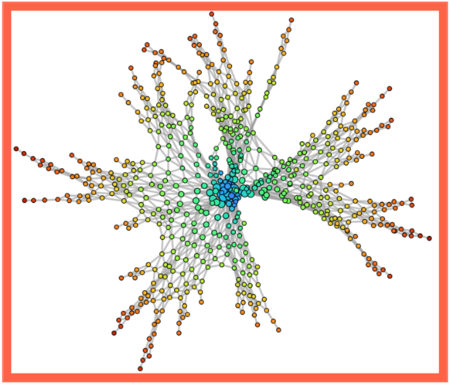
Knots as point clouds

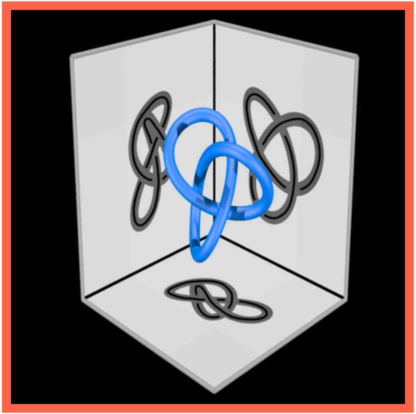
Or: Knots, data and TDA

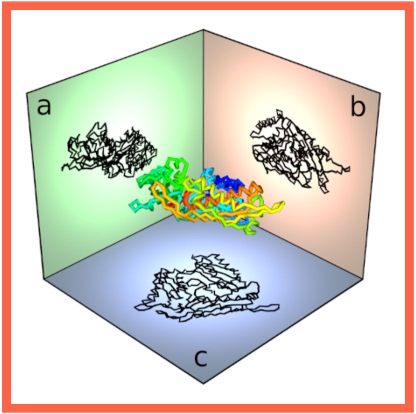
~~Accept~~Change what you cannot ~~change~~accept



I report on work of many people (Baldwin, Dłotko, Dowlin, Gurnari, Hajij, Kelomäki, Lacabanne, Levine, Levitt, Lidman, Sazdanovic, Vaz, Zhang, ...)

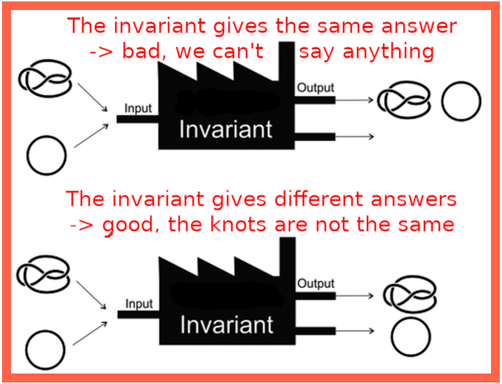
Goal: Use applied topology in quantum topology





- Knot = closed string (a circle S^1) in three spaces; link = multiple components
- Knots are studied by projections to the plane Shadows
- Knots/links are the basic building blocks of low dimensional manifolds

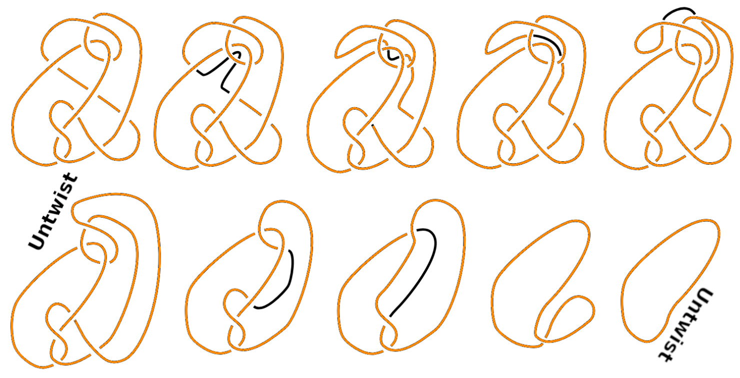
Goal: Use applied topology in quantum topology



- In math knot theory started in the early 20th century
- Topologists from ∼1900-1980 studied knots from the point of view of invariants from algebraic topology
- Problem The invariants obtained are not particularly "good"

Goal: Use applied topology in quantum topology

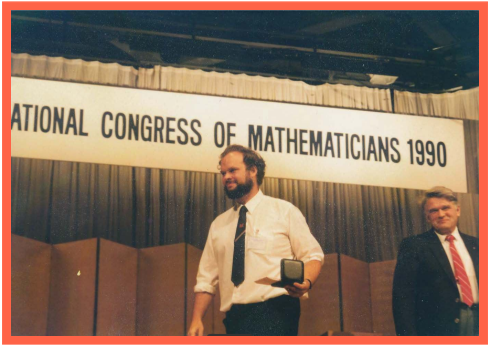
Even the unknotting problem is tricky



In general, knot theory was in need of new invariants
since the “standard invariants from algebraic topology”
(homology and friends)
are really not good for knots

- In
- To
  in
- Problem The invariants obtained are not particularly "good"

Goal: Use applied topology in quantum topology



- Kyoto 1990 Jones receives the fields medal (with Faddeev in the background)
- Quote "Jones discovered an astonishing relationship between von Neumann algebras and geometric topology. As a result, they found a new polynomial invariant for knots and links in 3-space."
- Today The focus is on the quantum knot invariants à la Jones

Goal: Use applied topology in quantum topology

1800s–early 1900s: proto invariants and the “classical topology” backbone

Date	Invariant (family)	Type/style	What it measures / why it matters
1833	**Linking number** (Gauss integral)	geometric ↔ algebraic topology	First robust link invariant; counts signed linking. ETSU Faculty +1
c. 1870s	**Crossing number** (as a concept in tabulation)	combinatorial/ diagrammatic	Not invariant of a *diagram* but of the isotopy class via minimization; key for classification culture.
c. 1870s	**Writhe** (diagram quantity)	combinatorial/ diagrammatic	Not a knot invariant under all Reidemeister moves, but essential in framed/regular isotopy settings.
1890s–1905	**Knot group** $\pi_1(S^3 \setminus K)$ via presentations	algebraic topology	The flagship “nonabelian” invariant; Wirtinger presentation becomes standard. School of Mathe... +1
c. 1900–1910	**Peripheral structure** (meridian/ longitude up to conjugacy)	algebraic topology	Upgrades the knot group; crucial for distinguishing knots with isomorphic groups.
c. 1910	**Colorings** / **Fox-type counting prototypes**	algebraic/combinatorial	Early forerunners of “quandle” thinking and finite-group counting invariants.

- Quote “Jones discovered an astonishing relationship between von Neumann algebras and geometric topology. As a result, they found a new polynomial invariant for knots and links in 3-space.”
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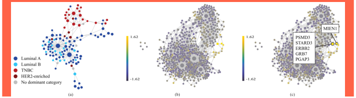
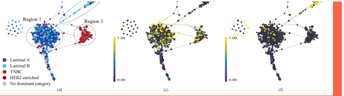
Goal: Use applied topology in quantum topology



Rostami, Z., Fooshee, D., Carlsson, G., & Subramaniam, S. (2025). Topological Data Analysis Reveals a Subgroup of Luminal B Breast Cancer. *IEEE Open Journal of Engineering in Medicine and Biology*, *6*, 465-471.

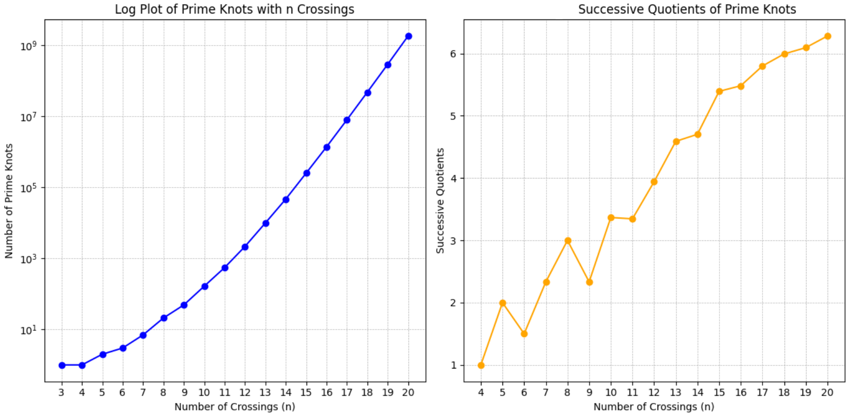


BIG DATA IN THEORETICAL MATH — This talk!

- Introduce new tools from applied algebraic topology and compare with other tools
- Focused on knot theory but the tools developed are not limited to knots or theoretical mathematics
- Use filtrations to analyze infinite data sets where representative sampling is impossible or impractical
- Analyze knot invariants and their relations

Goal: Use applied topology in quantum topology

Crucial

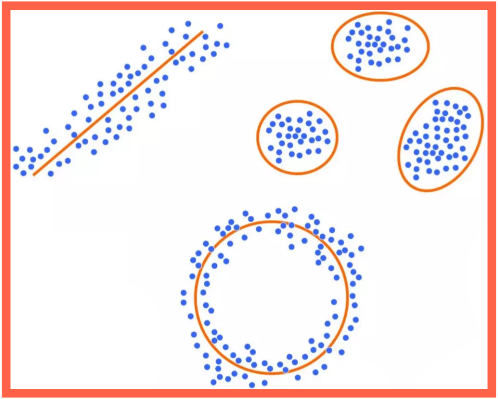


For this to work we need a lot of data; and we are lucky:

Ernst–Sumners ∼1987 The number of knots grows exponential

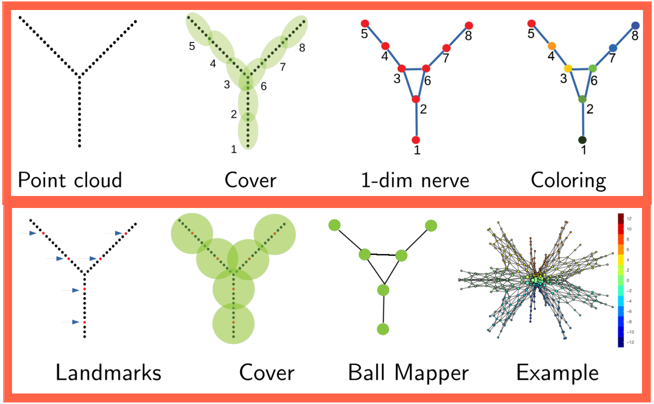
- Analyze knot invariants and their relations

Big data and quantum invariants



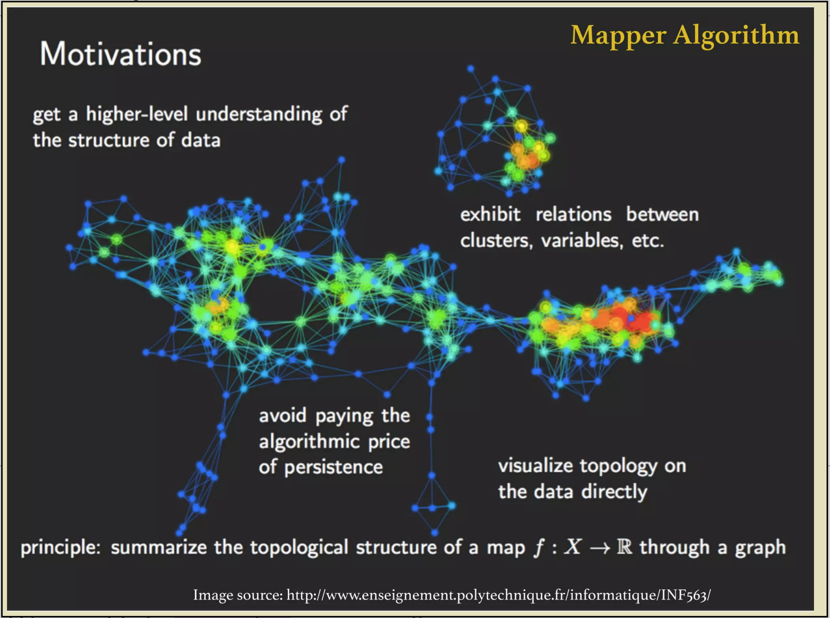
- TDA (topological data analysis) is the art of finding the shape of data
- Question What shape are quantum knot invariants?
- Question Can the shape measure how good they are?

Big data and quantum invariants



- (Ball) Mapper = a way to turn point clouds into a graph
- Coloring gives additional information
- We see this in examples momentarily

Big data and quantum invariants



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- We see this in examples momentarily

Big data and quantum invariants

Knots form point clouds!

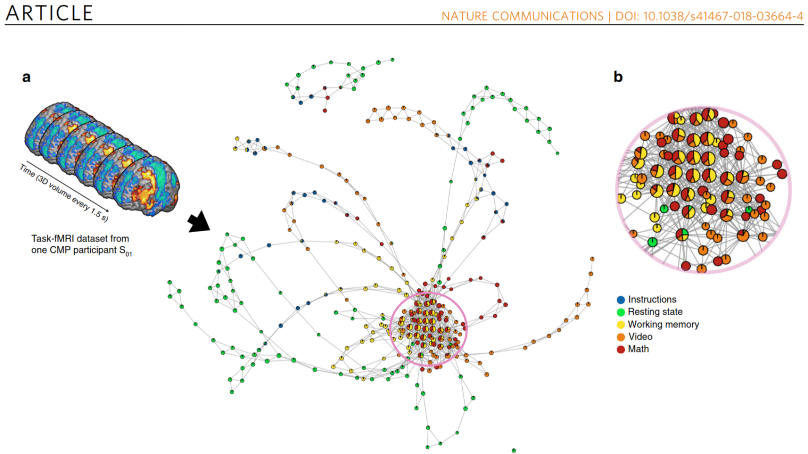
	q^{-3}	q^{-2}	q^{-1}	q^0	q^1	q^2	q^3	q^4	q^5	q^6	q^7
$J(0_1)$	0	0	0	1	0	0	0	0	0	0	0
$J(\mathrm{mir}(3_1))$	0	0	0	0	1	0	1	-1	0	0	0
$J(4_1)$	0	1	-1	1	-1	1	0	0	0	0	0
$J(\mathrm{mir}(5_1))$	0	0	0	0	0	1	0	1	-1	1	-1
$J(\mathrm{mir}(5_2))$	0	0	0	0	1	-1	2	-1	1	-1	0
$J(\mathrm{mir}(6_1))$	0	1	-1	2	-2	1	-1	1	0	0	0
$J(\mathrm{mir}(6_2))$	0	0	1	-1	2	-2	2	-2	1	0	0
$J(6_3)$	-1	2	-2	3	-2	2	-1	0	0	0	0

These are vectors in a 11d space

- Coloring gives additional information
- We see this in examples momentarily

Big

Example A mapper graph of the brain

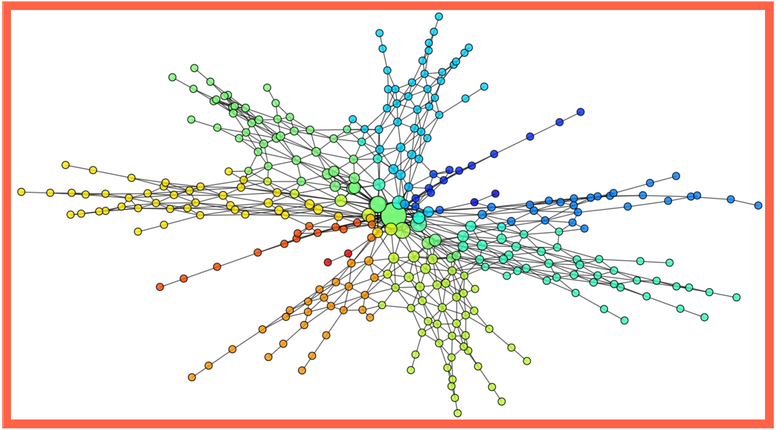


Idea Brain data is high dimensional and noisy $\Rightarrow$ Mapper helps!

Hmm... Jones polynomial data is high dimensional and noisy

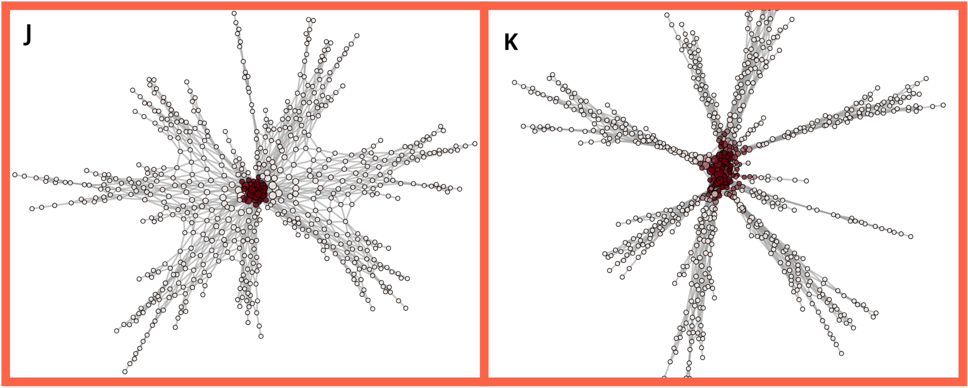
▶ We see this in examples momentarily

Big data and quantum invariants



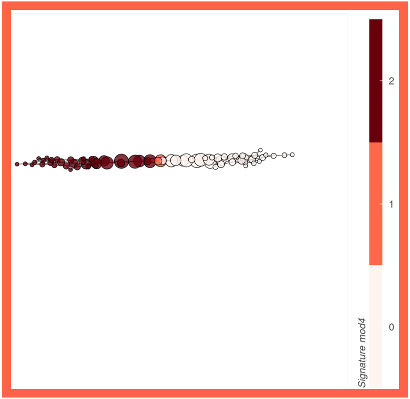
- Now live Ball mapper on knot data
- Play here https://dioscuri-tda.org/BallMapperKnots.html
  https://dustbringer.github.io/web–knot-invariant-comparison/

Big data and quantum invariants



- Above Jones and its categorification (homology version)
- Categorification "=" pushing things further apart
- Comparing the invariants shows that they are related

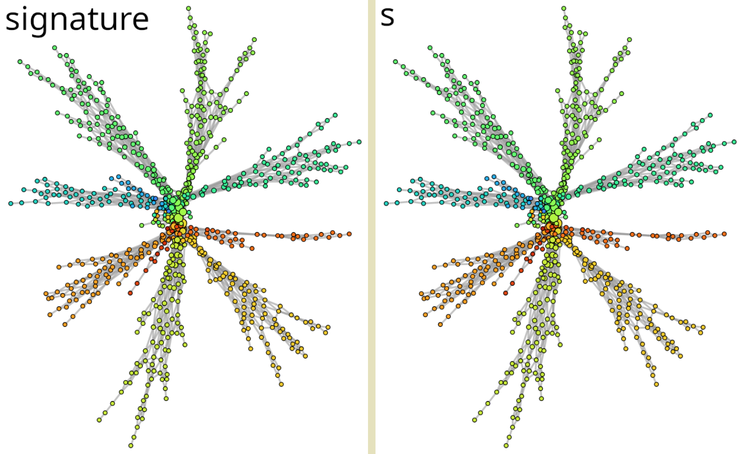
Big data and quantum invariants



- Above Coloring of the Alexander invariant with the signature mode 4
- Signature = a traditional knot invariant (from homology)
- The eye catching conjecture is then easy to prove

Big da

Example Signature = s invariant?

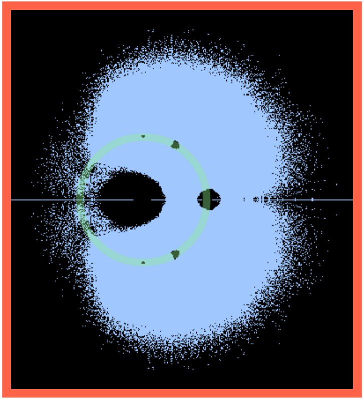


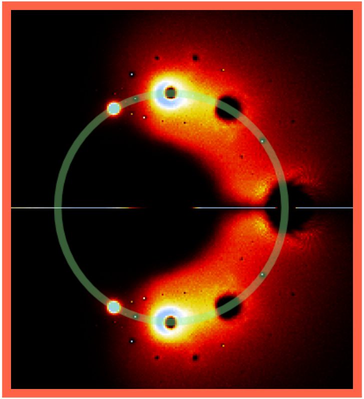
Not quite, but we were able to predict the correct statement

without knowing that this is true (or even what the invariants are)!

- A
- Si
- The eye catching conjecture is then easy to prove

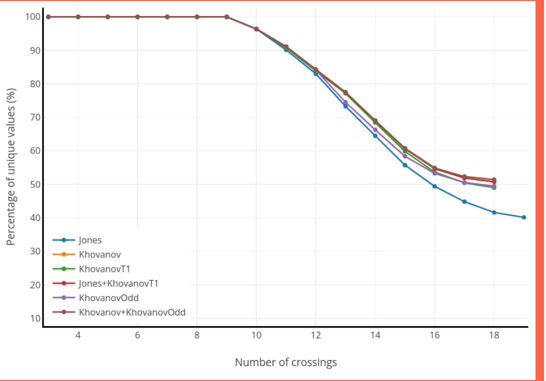
Big data and quantum invariants





- Above The roots of the Jones polynomials
- This is a very specific distribution
- Another task Compare the distribution of the polynomials

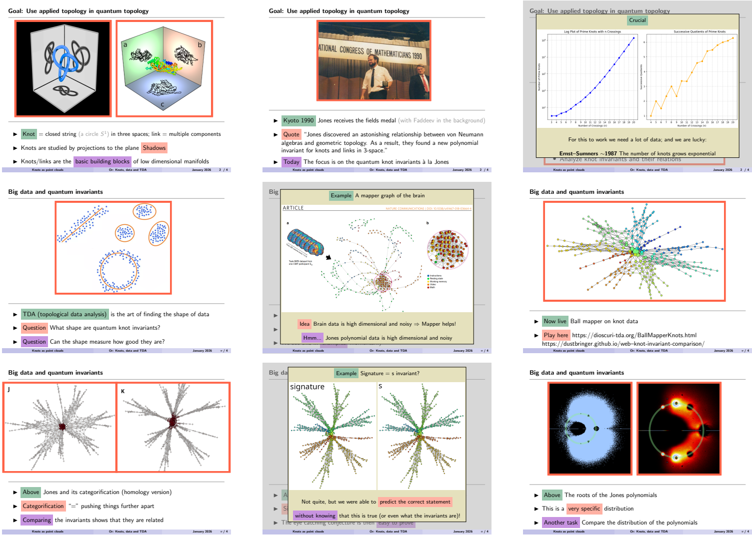
Big data and quantum invariants



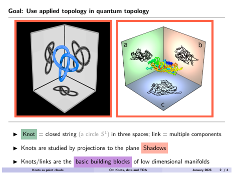
In Q3 2025, Restech provided over **18 million core-hours** of compute time through our **Katana Compute Cluster**, and your account was among the most active, with a total of **1,030,265 core-hours** consumed. The estimated/reference value is approximately **$41,211** .

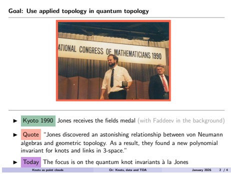
115 years
of computation
in just one Q

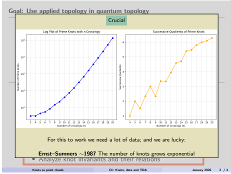
- Above The percentage of detectable knots (with Jones & friends)
- There are many more things on the website (the dataset is 5TB+)
- Play here https://dustbringer.github.io/web–knot-invariant-comparison/

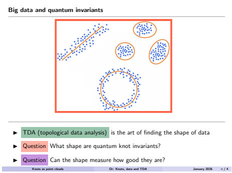


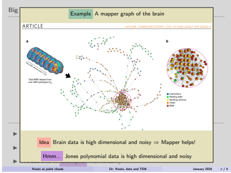
There is still much to do...

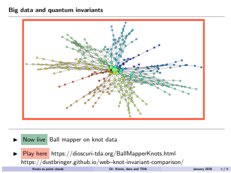


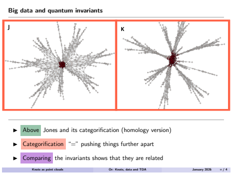


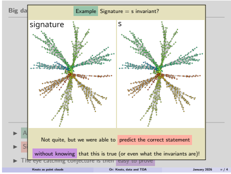


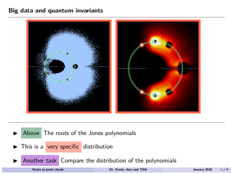












Thanks for your attention!